# Markets versus Mechanisms: Internet Appendix

**Raphael Boleslavsky**
University of Miami

**Christopher A. Hennessy**
London Business School

**David L. Kelly**
University of Miami

This file is the supplementary appendix to the paper "Markets versus Mechanisms," *Review of Financial Studies*, 2021.

## Appendix A. Proofs: Sections 2-4 of Main Text

This appendix gives concise proofs of the propositions in the paper. For a more detailed, step by step derivation of all results, please see Boleslavsky et al. (2020).

### A.1 Optimality of Uninformed Passivity: Mechanism Reliance

Consider a unilateral deviation by an uninformed outsider, given the mechanism has been left sitting. If he places a buy order, he creates two possible order vector possibilities, both off-path: one buy order in isolation or one buy order and one sell order. The uninformed outsider placing a buy order would make an expected loss since the market maker infers $\omega=1$ and sets $p=1$, whereas the fundamental value is only $1-q$. If an uninformed outsider instead places a sell order, he creates two possible order vector possibilities, one sell order or two sell orders, the former being on-path and the latter off-path (since the mechanism is left sitting and the expert does not exist). In the former case, the firm will implement the risky investment and the market maker sets price at $p=1-q$. In the latter case, the firm implements the safe investment

Published by Oxford University Press on behalf of The Society for Financial Studies 2021.
doi:10.1093/rfs/Sample Advance Access publication

and the market maker sets $p=1-c$. In both cases, the price equals the expected cash flow, resulting in zero profits.

### A.2 Optimality of Uninformed Passivity: Market Reliance

The uninformed investor confronts six possible combinations of type-0 expert, type-1 expert, no expert, and liquidity shock/not. With these scenarios in mind, consider first an uninformed outsider submitting a buy order. If the expert does not exist, the firm will implement the risky investment in response to the buy order, and price will be set to 1, but expected cash flow is only $1-q$, implying an expected loss of $q$. If the type-1 expert exists, the two buy orders will push price to the fundamental 1, resulting in zero profits. However, if the type-0 expert exists, the price will be set to 1 and the risky investment implemented absent a fully revealing liquidity shock, resulting in an uninformed loss of 1. Hence, submitting a buy order results in an expected loss.

Next, we consider an uninformed outsider submitting a sell order. If a liquidity shock occurs or a type-0 expert exists, there will be at least two sell orders, the firm will switch to safe, and there is zero profit. If no liquidity shock arrives, and the type-1 expert exists, the combination of a buy and a sell will induce the firm to select the risky investment and the price will be set at fundamental, $p=1$, resulting in zero profit. Finally, absent either a liquidity shock or an expert, there will be the single uninformed sell order. If the firm switches to the safe investment, there will be zero profit. If the firm implements the risky investment, a loss results since the updated belief $\chi(\vec{t})\geq q$ implies price $p=1-\chi(\vec{t})$ is less than expected cash flow $1-q$. Hence, a sell order results in an expected loss.

### A.3 Construction of High Market Informativeness Equilibrium

The construction of the posited zero-rent equilibrium proceeds as follows. If $u_0^*=0$, then each sell order $t$ must generate zero expected profit, otherwise the type-0 expert would have a profitable deviation. Equation (9) of the main text then implies that for all $t>0$, $\chi(\vec{t})=1$ and/or $\alpha(\vec{t})=1$, so that price equals fundamental value. But note, from the firm sequential rationality condition, Equation (7) of the main text, it follows that $\chi(\vec{t})=1$ implies $\alpha(\vec{t})=1$. Therefore, in any no-rent equilibrium, the posterior must be sufficiently negative to justify switching to the safe investment for any $t\in(0,1]$:

$$c\leq\frac{K\phi_0(t)+q}{K\phi_0(t)+1}. \tag{A1}$$

This implies that for all $t\in(0,1]$, the type-0 expert's trading density must satisfy:

$$\phi_0(t)\geq\frac{J-1}{K}. \tag{A2}$$

Since the trading density must integrate to 1 on the unit interval, a no-rent equilibrium cannot be sustained if $K<J-1$. Conversely, if $K\geq\overline{K}=J-1$, many feasible mixing densities exist which satisfy the preceding equation. That is, a multiplicity of payoff equivalent equilibria exist if $K\geq\overline{K}$.

### A.4 Construction of Low Market Informativeness Equilibrium

Consider an equilibrium in which $\alpha(\vec{t})=0$ for all $t\in(0,1]$. In this case, the type-0 expert's indifference condition is that for all $t\in[m,1]$,

$$m(1-q)(1-l)=t[1-\chi(\vec{t})](1-l). \tag{A3}$$

Substituting the market maker belief, equation (14) of the main text, into the preceding indifference condition, we find that

$$m(1-q)(1-l)=t\left[1-\frac{K\phi_0(t)+q}{K\phi_0(t)+1}\right](1-l)\Rightarrow\phi_0(t)=\frac{t-m}{Km}. \tag{A4}$$

Thus, in the posited equilibrium, the type-0 expert outsider exploits his private information by using a mixing density that increases linearly in the trade size $t$. To determine the minimum sell order $m$, note that $\phi_0(t)$ must integrate to 1. We have

$$\int_m^1\frac{t-m}{Km}dt=1\Rightarrow m=K+1-\sqrt{(K+1)^2-1}. \tag{A5}$$

Finally, since the market maker belief, Equation (14) of the main text, is increasing in $\phi_0$ which is itself increasing in $t$, we must verify that, as posited, the firm will find it optimal not to switch even if $t=1$, which demands belief $\chi(1)\leq c$. The preceding inequality holds only if $m\geq 1/J$, which itself holds only if $K\leq\underline{K}$, where

$$\underline{K}\equiv\frac{(J-1)^2}{2J}. \tag{A6}$$

### A.5 Construction of Intermediate Market Informativeness Equilibrium

With $\alpha(\vec{t})=0$ on the interval $[m,t']$, the type-0 expert's trading density can once again be derived from Equation (A4), with $\phi_0=(t-m)/Km$. At $t'=Jm$, $\chi(t')=c$ and further increases in $t'$ would be inconsistent with $\alpha(t')=0$. Therefore, on the interval $(t',1]$, the firm must mix between the safe and risky investments. For this mixing to be

sequentially rational, $\chi(\vec{t})=c$ on this interval. Combining this fact with Equations (14) and (18) of the main text, we conclude that

$$\text{for } t\in(Jm,1], \quad m(1-q)(1-l)=t(1-c)\left[1-\alpha(\vec{t})\right](1-l), \quad \text{and} \quad \frac{K\phi_0(t)+q}{K\phi_0(t)+1}=c. \tag{A7}$$

From the preceding equation, it follows that

$$\text{for } t\in(Jm,1], \quad \alpha(\vec{t})=1-\frac{Jm}{t}, \quad \text{and} \quad \phi_0(t)=\frac{J-1}{K}. \tag{A8}$$

Notice, since beliefs and prices are constant on this interval, the probability of the firm switching to the safe investment must increase in the size of the sell order to just offset the type-0 expert's temptation to submit larger orders. Once again, the fact that the expert's trading density $\phi_0(t)$ must integrate to 1 allows us to pin down the minimum sell order size:

$$\int_m^{Jm}\frac{t-m}{Km}dt+(1-Jm)(J-1)/K=1\Rightarrow m=\frac{2(J-K-1)}{J^2-1}. \tag{A9}$$

Finally, based on the preceding equation, we can verify the conjectured equilibrium is internally consistent. First, the type-0 expert makes a rent if $m>0$, which holds if and only if $K<J-1\equiv\overline{K}$. Second, the firm's mixed investment interval is nondegenerate if $Jm<1$, which holds if and only if $K>\underline{K}$ as defined above.

### A.6 Expected Cash Flow: Intermediate Market Informativeness

We provide a calculation for the ex ante expected cash flow in the case of intermediate market informativeness. Conditional on order vector $T$, the firm's expected cash flow is

$$(1-\chi(T))(1-\alpha(T))+\alpha(T)(1-c)=1-\chi(T)+\alpha(T)(\chi(T)-c).$$

Hence, expected cash flow is given by:

$$E[1-\chi(T)]+E[\alpha(T)(\chi(T)-c)],$$

where the expectation is taken with respect to the distribution of the equilibrium order flow vector. Because $1-\chi(T)$ is the probability of economic state one conditional on order vector $T$, the Law of Iterated Expectations implies that $E[1-\chi(T)]=1-q$. Next note that $\alpha(T)=0$ if a buy order arrives or the market is inactive, and hence, in these cases $\alpha(T)(\chi(T)-c)=0$. If instead a single sell order arrives, the firm either selects risky, $\alpha(T)=0$, or it mixes, $\alpha(T)\in(0,1)$. In the latter case, sequential rationality requires $\chi(T)=c$. Thus, whenever a single sell order arrives, $\alpha(T)(\chi(T)-c)=0$. Finally, under the remaining order flow configuration with two sell orders, $\chi(T)=1$ and $\alpha(T)=1$, and hence, $\alpha(T)(\chi(T)-c)=1-c$. Therefore,

$$E[\alpha(T)(\chi(T)-c)]=aql(1-c).$$

## Appendix B. Proof of Proposition 10, Optimal Search Friction

We prove the claims in Proposition 10 of the main text as a series of lemmas. We begin with the derivative of firm value given low informativeness with respect to $\pi$, Equation (36) of the main text, which determines the level of $\pi$ which maximizes firm value.

$$K_\pi < \underline{K} \Rightarrow V'(\pi) = qa(1-l)(1-c)\left[1 - JK\left(\frac{K(1-\pi)+1}{\sqrt{[K(1-\pi)+1]^2-1}} - 1\right)\right]. \tag{B1}$$

It is readily verified that $V'' < 0$ on this interval and that $V'$ tends to $-\infty$ as $\pi \uparrow 1$ on this interval. It follows that an interior solution results if $V'(0) > 0$, otherwise $\pi = 0$ is optimal:

**Lemma 1.** If $K \leq \underline{K}$, pure market reliance is optimal ($\pi^* = 0$) if switching difficulty is sufficiently high such that

$$\frac{1}{J} \leq K\left(\frac{K+1}{\sqrt{(K+1)^2-1}} - 1\right).$$

Otherwise, an interior optimum $\pi^*$ obtains which is continuously decreasing in the switching difficulty $J$.

Next, we consider optimal $\pi$ if $K \in (\underline{K}, \overline{K})$.

To begin, note that for any $K_\pi \in (\underline{K}, \overline{K})$, the intermediate informativeness market equilibrium would obtain resulting in the same firm value expression as in Equation (35) of the main text, but with $m_I$ replacing $m_L$ just as in the baseline model:

$$K_\pi \in (\underline{K}, \overline{K}) \Rightarrow V(\pi) = (1-q) + qa[\pi + (1-\pi)l](1-c) - am_I(K_\pi)q(1-q)(1-l).$$

Since $m_I < m_L$ it is apparent that if $K \in (\underline{K}, \overline{K})$, the firm would never want to choose $K_\pi \leq \underline{K}$, pushing the valuation into low informativeness territory. Further, we find that:

$$K_\pi \in (\underline{K}, \overline{K}) \Rightarrow V'(\pi) = qa(1-l)(1-c)\left[1 - \frac{2JK}{J^2-1}\right].$$

From the preceding equation we have the following lemma.

**Lemma 2.** If $K \in (\underline{K}, \overline{K})$, then pure market reliance ($\pi^* = 0$) is optimal if $K \in ((J^2-1)/2J, \overline{K})$. Otherwise $\pi^* = 1 - \underline{K}/K$.

Finally, consider $K \geq \overline{K}$. If such a firm chooses any $K_\pi \geq \overline{K}$ then the zero rent market equilibrium would obtain, implying a reservation value of zero for the expert participating in the mechanism. Moreover, so long as the expert existed, the firm would choose the optimal strategy in each economic state. But as in the zero rent equilibrium in the baseline model,

the firm would act suboptimally by switching to the safe investment if the expert did not exist but a liquidity shock were to hit. The implied firm value is then exactly as in the baseline model. That is,

$$K_\pi \geq \overline{K} \Rightarrow V(\pi) = V_{NR} = V^* - l(1-a)(c-q). \tag{B2}$$

It is readily verified the preceding value is higher than what the firm could obtain if it pushed $\pi$ high enough to switch to the intermediate informativeness valuation, which is higher than the low informativeness valuation. We thus have the following lemma.

**Lemma 3.** If $K \geq \overline{K}$ then pure market reliance is optimal ($\pi^* = 0$).

The preceding lemmas together prove Proposition 10 of the main text.

## Appendix C. Formal Derivation of the DRM

This section derives the direct revelation mechanism (DRM) from first principles, when the reservation value is exogenous. Such a derivation is needed because the firm has only limited commitment power. In particular, the requirement that the firm allocates the mechanism to the first willing agent and the firm's fiduciary responsibility to act optimally on its information (sequential rationality) both limit commitment power. In spite of the limited commitment, we show that given a sufficient bonding capability $B$, the firm can devise a mechanism in which the firm selects the first-best investment, while reducing the expert outsider's payoff to his reservation value.

The analysis in this section proceeds as follows. We first derive a set of constraints that are *necessary* for a mechanism to achieve higher ex ante firm value than can be achieved under market reliance. We next characterize conditions under which it is feasible to satisfy these necessary conditions, and then solve for the optimal mechanism(s) among those satisfying the necessary conditions.

Let $\underline{u}$ represent the expert's continuation payoff from rejecting the posted mechanism. In this section, we treat $\underline{u}$ as exogenous and assume that $\underline{u} > qu_0^*$. Note that an uninformed outsider's continuation payoff from rejecting the firm's mechanism offer is zero, since an uninformed outsider has no private information or market power.

Recall, fiduciary duty requires that the firm's behavior is sequentially rational. Since the firm cannot commit to future actions, the Revelation Principle does not apply directly. However, we establish an analogous result in Lemmas 4 and 6. Some formalities are first necessary. To this end, let $\chi_r$ be the firm's belief that the state is $\omega = 0$ following report

$r \in \{0,1\}$. As a normalization, let us label the reports so that $\chi_1 \leq q \leq \chi_0$.[1] Let $\rho_r$ be the probability that the firm selects the risky investment following report $r$. Let $x \in \{0,1\}$ denote the expert's participation decision, where $x=1$ represents the decision to participate.[2] Let $\gamma_\omega$ be the probability that the adviser sends report $r=1$ in economic state $\omega$. Finally, let $d$ be the probability that each uninformed outsider agrees to participate in the mechanism.

Any mechanism that outperforms market reliance must have certain properties. First, any such mechanism must be rejected by the uninformed outsiders and accepted by the expert outsider if he exists. After all, the uninformed outsiders are countably infinite, and the mechanism is assigned to the first willing agent. Thus, a mechanism that does not screen out the uninformed will almost surely be accepted by an uninformed outsider, and hence, cannot deliver useful information about the economic state. The firm would therefore watch the market for information, and both the firm and the market maker anticipate that the expert will be active in the market if he exists. Thus, offering a mechanism that fails to screen out incompetents cannot do better than market reliance. Following the same logic, any mechanism that fails to induce participation by the expert also cannot do better than market reliance. We have the following lemma.

**Lemma 4.** (Screening). If a mechanism delivers higher ex ante firm value than market reliance, then it must screen out uninformed agents and induce participation by the expert if he exists, $d=0$ and $x=1$.

Second, any mechanism that achieves higher ex ante firm value than market reliance has the property that it grants the expert real authority in the sense that $\rho_1=1$ and $\rho_0=0$. That is, in equilibrium the firm will follow the "recommendation" of its agent, implementing the risky investment with probability 1 (0) in response to report $r=1$ ($r=0$). To see why this must be the case, recall first that report-contingent beliefs are such that $\chi_1 \leq q \leq \chi_0$. Sequential rationality therefore demands $\rho_1=1$. Next, we consider why any mechanism that is value-increasing relative to market reliance must satisfy $\rho_0=0$. To begin, note that any mechanism that induces participation by the expert, as is necessary, features an expected wage bill no less than $a\underline{u} > aqu_0^*$. This exceeds the adverse selection cost under market reliance. Therefore, any value-increasing mechanism must lead to a strict increase in expected cash flow

[1] The Law of Iterated Expectations requires $\Pr(r=0)\chi_0 + \Pr(r=1)\chi_1 = \Pr(\omega=0) = q$. Therefore one posterior belief must be weakly smaller than the prior and the other weakly larger.

[2] For brevity, we abstract from mixing by the expert in his participation decision in this section. Section 5.2 of the main text considers an extension that is formally equivalent to a setting in which the expert mixes in the participation decision.

relative to market reliance. With this in mind, we consider expected cash flow under a mechanism featuring $\rho_0 \in (0,1]$. If $\rho_0 = 1$, the firm always implements the risky investment, and expected cash flow is $1-q$, which is less than the expected cash flow under market reliance.[3] If instead $\rho_0 \in (0,1)$, sequential rationality requires the firm to be indifferent between $S$ and $R$ following $r=0$. But note, mixing implies expected cash flow is the same as if the firm were to always choose the risky investment. To see this formally, note that the firm is willing to mix only if $\chi_0 = c$, and hence:

$$\begin{aligned} E[\varphi] &= \Pr(r=0)[\rho_0(1-\chi_0)+(1-\rho_0)(1-c)]+\Pr(r=1)(1-\chi_1) \\ &= \Pr(r=0)[\rho_0(1-\chi_0)+(1-\rho_0)(1-\chi_0)]+\Pr(r=1)(1-\chi_1) \\ &= \Pr(r=0)(1-\chi_0)+\Pr(r=1)(1-\chi_1) = 1-q. \end{aligned}$$

The last line above follows from the Law of Iterated Expectations. We thus have the following lemma.

**Lemma 5.** (Delegated decision). If a mechanism delivers higher ex ante firm value than market reliance, then it must delegate the decision to the expert, $\rho_0 = 0$ and $\rho_1 = 1$.

Third, any mechanism that achieves higher ex ante value than under market reliance induces the expert to report truthfully with probability 1. After all, if it is sequentially rational for the firm to follow the expert's advice, with $\rho_0 = 0$ and $\rho_1 = 1$ (see preceding lemma) then it must be that $\chi_0 \geq c > q \geq \chi_1$. Therefore, the expert must tell the truth with positive probability (i.e., he cannot strictly prefer to lie): $\gamma_1 > 0$ and $\gamma_0 < 1$. These conditions imply two constraints on wages. First, to ensure $\gamma_1 > 0$, it must be that $w_{11} \geq w_{01-c}$. Second, to ensure $\gamma_0 < 1$, it must be that $w_{01-c} \geq w_{10}$. Furthermore, as we show in Appendix A.6, in any mechanism that delivers a higher payoff than market reliance (consistent with Lemma 4 and 5), these two constraints on wages hold with strict inequality, and so the expert strictly prefers to report truthfully.

**Lemma 6.** (Truthful reporting). If a mechanism screens out uninformed agents and induces participation by the expert (as in Lemma 4) and delegates the decision to the expert (as in Lemma 5), then the expert's unique sequentially rational strategy is to report truthfully with probability 1, $\gamma_0 = 0$ and $\gamma_1 = 1$.

Lemma 5 allows us to focus on mechanisms in which the adviser's wage depends only on the firm's terminal cash flow, not on his report. To see this, note that if the adviser reports $r=1$, the firm implements

[3] See equations 16 and 21 of the main text.

the risky investment with probability 1 which implies wage $w_{11-c}$ is irrelevant. Similarly, if the adviser reports $r=0$, the firm implements the safe investment which implies wages $w_{00}$ and $w_{01}$ are irrelevant. We thus need only focus on wages $\{w_{01-c}, w_{11}, w_{10}\}$, which can be written as a function only of the realized cash flow. Therefore, in what follows we drop the first subscript (the report) from the agent's wage.

Therefore, Lemmas 4–6, along with the agents' liability constraint, imply that any mechanism delivering higher ex ante firm value than market reliance must satisfy constraints (SC1), (SC2), (PC), and (BOND) from Section 2 of the main text.

Constraint (SC0) ensures that an uninformed outsider prefers to reject the mechanism, rather than accept and report $r=0$. Similarly, (SC1) rules out an uninformed agent participating and reporting $r=1$.[4] Constraint (PC) ensures that the expert outsider is willing to participate in the mechanism if he indeed exists, anticipating that he will report the state truthfully; from Lemma 6 we know that the expert's only sequentially rational strategy is truthful reporting (with probability 1) in any mechanism that delivers a higher payoff than market reliance. The constraints in (BOND) reflect the expert's limited liability. We refer to the set of constraints as $\mathcal{S}$. Because (SC0), (SC1), and (PC) are imposed by the mechanism's need to screen out uninformed outsiders and screen in the expert, we refer to $\mathcal{S}$ as the *screening constraints*. If the screening constraints are mutually consistent, we say that *screening is feasible*, and we refer to a mechanism that satisfies $\mathcal{S}$ as a *feasible* mechanism.[5]

To meet the expert's participation constraint, the firm needs to ensure that a particular linear combination of $w_{1-c}$ and $w_1$ is sufficiently large. However, increasing $w_{1-c}$ makes it more attractive for an uninformed agent to accept and report $\omega=0$, while increasing $w_1$ makes it more attractive for an uninformed agent to accept and report $\omega=1$. The temptation for an uninformed agent to report $\omega=1$ can be offset by reducing $w_0$, thereby generating a punishment for incorrectly reporting that the state is good. However, the firm's ability to punish is restricted by the agent's limited liability, and so screening is not always feasible as shown in the following proposition.

**Proposition 1.** (Feasible screening). If the expert's reservation value $\underline{u} > qB$, then screening is infeasible and every mechanism does no better than market reliance.

[4] (SC0) and (SC1) also ensure that an uninformed agent would rather reject than accept and then report randomly.

[5] Note we define feasibility as existence of a mechanism which potentially delivers ex ante value in excess of market reliance.

We now find the optimal mechanism assuming liability is large enough that screening is feasible. The firm's objective is to maximize the ex ante value of a share (or equivalently, total firm value) subject to $\mathcal{S}$. In any feasible mechanism, ex ante firm value is

$$(1-a)(1-q)+a[(1-q)+q(1-c)]-a[qw_{1-c}+(1-q)w_1].$$

The first term reflects the fact that if no expert exists, the firm will implement the risky investment, with expected cash flow $1-q$. The second term is the firm's expected cash flow if the expert exists, with Lemmas 5 and 6 informing us that any feasible mechanism has the property that the firm selects the correct investment in each economic state if the expert exists. The final term is the expected wage bill.[6]

Summarizing, we have proven Proposition 1 of the main text, which we state formally in Proposition 2.

**Proposition 2.** (Optimality). If $\underline{u} \leq qB$, then a feasible mechanism is optimal if and only if (PC) holds with equality. In every optimal mechanism, project selection is first-best and ex ante firm value is

$$\begin{aligned} V_{DRM} &= (1-a)(1-q)+a[(1-q)+q(1-c)-\underline{u}] \\ &= 1-q+aq(1-c)-a\underline{u} \\ &= V^*-a\underline{u}. \end{aligned} \tag{C1}$$

The following mechanism is feasible and optimal whenever $\underline{u} \leq qB$:

$$(w_0, w_{1-c}, w_1) = \left(-B, 0, \frac{\underline{u}}{1-q}\right).$$

## Appendix D. Proofs from Appendix A.6

**Proof.** (Lemma 5) (i) $\rho_1 = 1$ follows from $\chi_1 \leq q$ and the firm's sequential rationality, Equation (7) of the main text. (ii) Note that any mechanism that beats remaining unadvised must induce the expert to participate and screen out uninformed agents and has $\rho_1 = 1$. Hence, the expected payoff to the firm in any such mechanism is

$$\begin{aligned} &(1-a)(1-q)+a\Pr(r=0)\{(1-\chi_0)\rho_0+(1-\rho_0)(1-c))\}+ \\ &\qquad a\Pr(r=1)(1-\chi_1)-aU, \end{aligned}$$

[6] Note that Lemmas 5 and 6 imply that the firm always selects the correct action in each state whenever the expert exists.

where $U \geq \underline{u}$ is the expert's expected wage. Suppose $\rho_0 = 1$. Using the Law of Iterated Expectations, the firm's payoff simplifies,

$$(1-a)(1-q)+a\Pr(r=0)(1-\chi_0)+a\Pr(r=1)(1-\chi_1)-aU=$$
$$(1-a)(1-q)+a(1-q)-aU=1-q-aU.$$

Therefore, $\rho_0 = 1$ is inferior to market reliance.

Suppose $\rho_0 \in (0,1)$. Sequential rationality by the firm, Equation (7) of the main text, requires $\chi_0 = c$, and hence, the firm's payoff simplifies to

$$(1-a)(1-q)+a\Pr(r=0)\{(1-\chi_0)\rho_0+(1-\rho_0)(1-\chi_0))\}+a\Pr(r=1)(1-\chi_1)-aU=$$
$$(1-a)(1-q)+a\Pr(r=0)(1-\chi_0)+a\Pr(r=1)(1-\chi_1)-aU=$$
$$(1-a)(1-q)+a(1-q)-aU=$$
$$(1-q)-aU.$$

Note that the transition from the second to the third lines uses the Law of Iterated Expectations. Note that $U \geq \underline{u} > qu_0^*$. Thus, the cost of offering the mechanism $aU$ exceeds the adverse selection cost under market reliance, $aqu_0^*$. Finally, consider the firm's cash flow under market reliance. If $K < \overline{K}$, then Equation (20) of the main text gives the firm's cash flow of $1-q+aql(1-c) > 1-q$. If $K > \overline{K}$, then from Equation (16) of the main text, it is $1-q+l(1-c)(1-a)\left(K/(1-l)-\overline{K}\right) > 1-q$. Thus, the firm's expected cash flow is larger under market reliance. Simultaneously, the adverse selection cost under market reliance is smaller than the expected wage bill under the mechanism. Therefore, $0 < \rho_0 < 1$, is inferior to market reliance. ■

**Proof.** (Lemma 6). Claim 1: *If a mechanism delivers the firm a higher expected payoff than market reliance, then it cannot be the case that* $\gamma_0 = \gamma_1 = 1$. If $\gamma_0 = \gamma_1 = 1$, then $\chi_1 = q$ and any value of $\chi_0$ is consistent with Bayes' rule. Because the expert always reports $r = 1$ in equilibrium, the firm always implements the risky action, and hence expected firm value is $1-q-aU$, where $U \geq \underline{u}$ is the expert's expected wage. This is smaller than the expected firm value under market reliance, as shown in the proof of Lemma 5. Claim 2: *If a mechanism delivers the firm a higher expected payoff than market reliance, then* $w_{11} \geq w_{01-c}$ *and* $w_{01-c} \geq w_{10}$. From Lemma 5, $\rho_0 = 0$, and hence, $\chi_0 \geq c$. From Bayes' rule,

$$\chi_0 = \frac{q(1-\gamma_0)}{q(1-\gamma_0)+(1-q)(1-\gamma_1)}.$$

From Claim 1, $\chi_0$ is well defined. Hence,

$$\chi_0 \geq c \iff q(1-c)\gamma_0 + c - q \leq c(1-q)\gamma_1. \tag{D1}$$

First we show $w_{11} \geq w_{01-c}$. Note that

$$\begin{aligned} q(1-c)\gamma_0 + c - q \leq c(1-q)\gamma_1 \Rightarrow \\ c - q \leq c(1-q)\gamma_1 \Rightarrow \\ \gamma_1 \geq \frac{c-q}{c(1-q)} > 0. \end{aligned}$$

Thus, $\gamma_1 > 0$, which implies the expert must report truthfully in state 1 with positive probability. Thus, the expert's expected payoff of reporting truthfully in state 1 must be at least as large as his expected payoff of lying, and hence $w_{11} \geq w_{01-c}$.

Next, we show $w_{01-c} \geq w_{10}$. Suppose that $\gamma_0 = 1$. Substituting into (D1),

$$q(1-c) + c - q \leq c(1-q)\gamma_1 \Rightarrow \gamma_1 \geq 1.$$

Hence, $\gamma_0 = 1$ implies $\gamma_1 = 1$, contradicting Claim 1. Hence, $\gamma_0 < 1$, which implies that the expert must report truthfully in state 0 with positive probability. Thus, the expert's expected payoff of reporting truthfully in state 0 must be at least as large as his expected payoff of lying, and hence $w_{01-c} \geq w_{10}$.

Claim 3: *If a mechanism delivers the firm a higher expected payoff than market reliance, then* $w_{11} > w_{01-c}$ *and* $w_{01-c} > w_{10}$. From Lemma 4, any mechanism which achieves higher value than market reliance screens out uninformed outsiders and requires participation of the expert. These constraints are

$$w_{01-c} \leq 0 \tag{SC0}$$

$$qw_{10} + (1-q)w_{11} \leq 0 \tag{SC1}$$

$$q[\gamma_0 w_{10} + (1-\gamma_0)w_{01-c}] + (1-q)[\gamma_1 w_{11} + (1-\gamma_1)w_{01-c}]. \geq \underline{u} \tag{PC}$$

Constraint (SC0) ensures that uninformed outsiders prefer to reject the mechanism over accepting and reporting $r = 0$, (SC1) ensures that uninformed outsiders prefer to reject the mechanism over accepting and reporting $r = 1$, and (PC) ensures that an expert prefers to participate (if he exists).

Next, note that Claim 2 ensures $w_{11} \geq w_{01-c}$. Therefore, either $w_{11} > w_{01-c}$, which implies $\gamma_1 = 1$, or $w_{11} = w_{01-c}$. In either case, (PC) reduces to

$$q[\gamma_0 w_{10} + (1-\gamma_0)w_{0,1-c}] + (1-q)w_{11} \geq \underline{u}$$

Analogously, either $\gamma_0 = 1$ or $w_{01-c} = w_{10}$ in which case (PC) reduces further to

$$qw_{01-c} + (1-q)w_{11} \geq \underline{u}. \tag{PC$'$}$$

Hence:

$$w_{11} \geq \frac{\underline{u}}{1-q} - \frac{q}{1-q} w_{01-c} > 0,$$

where the last inequality follows because $\underline{u} > 0$ and $w_{01-c} \leq 0$. Hence, $w_{11} > 0 \geq w_{01-c}$.

Note further that subtracting (SC1) from (PC′) yields

$$qw_{01-c} + (1-q)w_{11} - (qw_{10} + (1-q)w_{11}) \geq \underline{u} \Rightarrow w_{01-c} \geq w_{10} + \frac{\underline{u}}{q} \Rightarrow w_{01-c} > w_{10},$$

where the last inequality follows from $\underline{u} > 0$.

Claim 4: *The expert's unique sequentially rational reporting strategy is* $\gamma_0 = 0$ *and* $\gamma_1 = 1$. Follows immediately from Claim 3. ■

**Proof.** (Proposition 1). We show that (SC0), (SC1), (PC), and (BOND) imply $\underline{u} \leq qB$. Subtracting (SC0) from (PC′) yields $(1-q)w_1 \geq \underline{u}$. Substituting into (SC1) we find that $w_0 \leq -\underline{u}/q$. Hence, (BOND) implies that $\underline{u}/q \leq B$, and hence $\underline{u} \leq qB$. ■

**Proof.** (Proposition 1, main text). In the text, we argued that in any feasible mechanism, expected firm value is

$$(1-a)(1-q) + a[(1-q) + q(1-c)] - a[qw_{1-c} + (1-q)w_1].$$

Thus, the firm would like to minimize expected compensation, $qw_{1-c} + (1-q)w_1$, but (PC) requires $qw_{1-c} + (1-q)w_1 \geq \underline{u}$. Hence, any feasible mechanism in which (PC) holds with equality is optimal, yielding payoff

$$(1-a)(1-q) + a[(1-q) + q(1-c)] - a\underline{u} = 1 - q + aq(1-c) - a\underline{u}.$$

Via direct substitution, it is readily verified that the proposed mechanism is feasible and optimal if $\underline{u} \leq qB$. ■

## Appendix E. Analysis of Alternative Technology: Safe Investment Is Optimal Given Prior Beliefs

We now consider an alternative real technology, one in which the safe investment is optimal given only prior information, $q > c$. The firm only switches to the risky investment if sufficient positive information about the economic state is revealed. With this in mind, we make a second change to the baseline model and assume now that there is zero noise selling. Rather, with probability $l > 0$ there exists a fixed $b > 0$ of liquidity buying of the company's stock.[7] This noise buying allows for

[7] Assuming liquidity demand is either $b$ or zero simplifies the exposition.

the possibility that an expert can make gains when buying on positive information. Finally, we assume as in section 6.2 of the paper that a search friction exists. In particular, the expert views the mechanism with some exogenous probability $\pi \in (0,1)$.

The existence of a search friction is a necessary condition for our main force to be operative given $q > c$, so that markets have the potential to dominate mechanisms. To see this, consider that with zero search friction, the market maker and firm would believe that after a mechanism has been posted and left sitting, no expert exists. But if no expert exists, there will be no updating based on order flow in the market. Therefore, the firm would optimally stay with the default safe investment, and so a deviating expert would stand to make zero trading gain. Therefore, for an expert to have a nonzero deviation payoff when $q > c$, a search friction must be present.

With a search friction, if the mechanism is not accepted, the firm and market maker are uncertain whether (a) the expert does not exist or (b) the expert did not see the mechanism. Thus, when the mechanism is posted, but not accepted, the market maker and firm update that the expert is less likely to exist, but they do not eliminate this possibility completely. Indeed, they infer that the expert may be in the market with a positive probability, but this probability is smaller than the prior belief. Thus, if the informed trader deviates and does not accept the mechanism, the firm does not automatically stay with the safe investment. Rather, under parametric conditions derived below, the firm optimally switches to the risky investment if a single buy order arrives after a mechanism has been posted and not taken.

In particular, we will prove below that

$$c > \frac{(1-a)lq}{a(1-\pi)(1-q)(1-l)+(1-a)l} \Rightarrow V_{MKT} > V_{DRM}. \tag{E1}$$

That is, even under a flipped real-technology where the safe investment is the default, the firm will find market reliance optimal if the market is sufficiently informative, with $a$ sufficiently high and/or $l$ is sufficiently low, provided there is a search friction. Furthermore, when $a$ is sufficiently close to 1 or $l$ to 0, the level of search friction that is required for this result is also close to 0.

### E.7 Market Reliance

Consider first market reliance. Since no noise selling exists, the expert stands to make zero trading gain by selling on negative information. Thus, we posit that the type-0 expert plays a mixed strategy, placing a sell order on the support $(0,1]$. Any such trade reveals the bad state and

so the firm stays with the safe investment.[8] The type-1 expert buys $b$ shares, using the potential existence of a noise buy of size $b$ as cover. In order for such a buy order to yield strictly positive expected profits, the firm must revise its beliefs sufficiently to induce a switch to the risky investment. In particular, now letting $\chi$ denote the revised belief that the state is bad after receiving a single *buy* order of size $b$, a switch to the risky investment is optimal iff

$$c \geq \chi = \frac{(1-a)lq}{a(1-q)(1-l)+(1-a)l}. \tag{E2}$$

For the remainder of the analysis it is assumed that the prior inequality is strict so that indeed switching to the risky investment is optimal upon the arrival of a single buy order.[9]

The expert makes trading gains here if the expert exists, the state is good, and no liquidity buy reveals the true state. In this case, the expert buys $b$ shares at price $1-\chi$ against a fundamental value of 1. Thus, evaluated *ex ante*, expected expert trading gains are

$$a(1-q)(1-l)b\chi. \tag{E3}$$

As in the baseline model we assume that adverse selection costs (expert trading gains) are borne by the original shareholders.

Next, we consider the expected cash flow of the firm. The first-best cash flow results from staying with the safe investment if the expert does not exist, switching to risky if the type-1 expert exists, and staying with the safe investment if the type-0 expert exists. The implied first-best expected cash flow (and firm value) is

$$V^* \equiv (1-a)(1-c)+a[(1-q)1+q(1-c)]. \tag{E4}$$

If the expert exists, then given (E2) the market-reliant firm implements the first-best described above, switching to risky if the state is good and staying with safe if the state is bad. However, if the expert does not exist, the market-reliant firm's cash flows are less than the first-best described above. In particular, if the expert does not exist, the firm implements the safe investment if no liquidity buy arrives, but incorrectly switches to the risky investment if a liquidity buy does arrive, resulting in a lower cash flow on average. The implied expected cash flow is

$$(1-a)[(1-l)(1-c)+l(1-q)]+a[(1-q)1+q(1-c)]. \tag{E5}$$

Subtracting the asymmetric information costs (E3) from the cash flows implies the market-reliant firm value is

$$V_{MKT} = V^* - (1-a)l(q-c) - a(1-q)(1-l)b\chi. \tag{E6}$$

[8] This is consistent with the corresponding assumption in the body of the paper, regarding the trading strategy of the type-1 expert.

[9] If (E2) fails to hold, the market does not yield any actionable information and so mechanisms trivially dominate markets.

### E.8 Mechanism Reliance

To begin, note that the mechanism design program is identical to that described in Subsection 3.1, except that the expert's outside option value will change, of course. With this in mind, consider now mechanism reliance, and the payoffs to an expert who has failed to see the mechanism posted, or an expert who has simply decided to leave a posted mechanism sitting (an off-path decision). Since no liquidity selling exists, the expert stands to make zero trading gain selling on negative information. Thus, we again posit that the type-0 expert here plays a mixed strategy, placing sell order on the support $(0,1]$. The firm then learns the economic state and stays with the safe investment. The type-1 expert buys $b$ shares, using the potential existence of a noise buy of size $b$ as cover. For such a buy order to yield strictly positive expected profits, the firm must revise beliefs sufficiently to induce a switch to the risky investment.

Letting $\widetilde{\chi}$ denote the updated belief that the state is bad after a mechanism has been left sitting and the arrival of a single buy order of size $b$, the firm optimally switches to the risky investment iff:

$$c \geq \widetilde{\chi} = \frac{(1-a)lq}{a(1-\pi)(1-q)(1-l)+(1-a)l}. \tag{E7}$$

Note that the right-hand side of (E7) is continuous and approaches $q$ as $\pi$ approaches 1. Because $c > q$, condition (E7) requires that $\pi < 1$, that is, a search friction is needed. However, when $a$ approaches 1, or l approaches 0, (E7) holds for any $\pi < 1$. Thus, if the market is sufficiently informative, even an infinitesimal search friction is consistent with (E7).

For the remainder of the analysis we assume the inequality (E7) is strict, so that indeed the firm optimally switches to the risky investment upon the arrival of a single buy order, after a mechanism has been posted and left sitting. If (E7) fails to hold, the expert has an outside option of zero since the firm would implement the safe investment after the arrival of a single buy order.

The key force in the paper is that posting a mechanism reduces price impact. In the alternative technology considered here, this force remains. In particular,

$$\pi > 0 \Rightarrow \widetilde{\chi} > \chi. \tag{E8}$$

That is, the posting of a mechanism here allows the type-1 expert to buy at a lower price (provided no revealing liquidity buy arrives).

Consider now the expected payoff accruing to the expert. By entering the market (either because he did not see the mechanism or because he deliberately did not accept it), the expert can trade with smaller price impact. Provided the state is good and no liquidity shock arrives, the expert buys $b$ shares at price $1-\widetilde{\chi}$ against a fundamental value of 1.

By entering the market rather than the mechanism, the expert expects trading gain

$$(1-q)(1-l)b\widetilde{\chi}. \tag{E9}$$

But note that if the expert observes the mechanism, he would only accept it if he were paid a wage equal to this expected trading gain, since he could always choose not to accept the mechanism and make this gain by entering the market.

Thus, from an ex ante perspective, if the firm offers the mechanism, it must give up this trading gain to the expert, either in the form of an adverse selection cost or a wage, whenever the expert exists. Thus, the expected wage bill/adverse selection cost under mechanism reliance is

$$a(1-q)(1-l)b\widetilde{\chi} > a(1-q)(1-l)b\chi, \tag{E10}$$

where the term on the right side of the inequality represents the adverse selection cost under market reliance. As in the main model, offering the mechanism increases the expert's trading gain from rejecting it, which increases his outside option.

Consider now expected cash flow of the mechanism-reliant firm. If the expert does not exist, the mechanism is of course left sitting, so that the firm implements the safe investment if no liquidity buy arrives, but incorrectly switches to the risky investment if a liquidity buy does arrive. If the expert exists and observes the mechanism, the mechanism-reliant firm implements the first-best described above, switching to risky if the state is good and staying with safe if the state is bad. Finally, if the expert exists but does not observe the mechanism, the expert's trading still causes the firm to implement the optimal state-contingent investment policy. Thus, expected cash flow for the mechanism-reliant firm is just equal to that of the market-reliant firm, as given in Equation (E5). Thus, we have established that

$$\begin{aligned} c \quad &\geq \quad \widetilde{\chi} = \frac{(1-a)lq}{a(1-\pi)(1-q)(1-l)+(1-a)l} \\ &\Rightarrow \quad V_{DRM} = V^* - (1-a)l(q-c) - a(1-q)(1-l)b\widetilde{\chi} < V_{MKT}. \end{aligned}$$

**References**

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