Dynamic Asset-Backed Security Design

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Abstract

Borrowers obtain liquidity by issuing securities backed by the current period payoff and resale price of a long-lived collateral asset, and they are privately informed about the payoff distribution. Asset price can be self-fulfilling: a higher asset price lowers adverse selection and allows borrowers to raise greater funding, which makes the asset more valuable, leading to multiple equilibria. Optimal security design eliminates multiple equilibria, improves welfare, and can be implemented as a repo contract. Persistent adverse selection lowers debt funding, generates volatility in asset prices and exacerbates credit crunches. The theory demonstrates the role of asset-backed securities on stability of market-based financial systems.

Keywords: Liquidity; Dynamic Price Feedback; Intertemporal Coordination; Security Design; Multiple Equilibria; Self-fulfilling Prices; Financial Fragility; Haircut; Repo; Repo Runs; Asset-Backed Security; Collateral; Limited Commitment; Adverse Selection; Non-Bank Based Financial Intermediation; NBFI; Market-Based Financial Intermediation.

JEL classification: G10, G01

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1 Introduction

In this paper, we propose a theory of security design when the underlying asset is long-lived and traded, and both the current period payoff and the resale price of the asset are used to back the optimally chosen securities. There is asymmetric information about asset quality: asset owners are better informed about the current period payoff and this information advantage might be short-lived or persistent. There are also gains from trade in the style of Lagos and Wright (2005): asset-backed securities are used as a means of payment for asset owners to fund some outside consumption or investment opportunity. In our model, due to its collateral role, the entire cum-dividend value of the asset creates gains from trade. Consequently, the severity of the adverse selection problem depends on the cum-dividend value of the asset, which activates price feedback: a higher asset price in the future lowers the adverse selection today which allows for more asset-backed security sales today, which in turn justifies the higher future asset price. In the model, security design and asset price are *jointly* determined in equilibrium. In this economic setting, we find that when the set of available securities is restricted, there are multiple equilibria with destabilizing self-fulfilling prices. Optimal security design eliminates multiplicity and leads to a unique Pareto-optimal equilibrium where the resale price of the asset is high and the sale of securities backed by the asset raises more funding.

The price feedback mechanism in our paper is empirically relevant because an increasing number of markets (exchange or over-the-counter) are being created to trade a variety of financial assets. Resale prices of these financial assets become collateralizable and constitute an important component of the collateral value for borrowing obligations. Financial institutions are becoming more dependent on markets to assess the collateral values when intermediating capital flows.[1] For example, short-term asset-backed borrowing facilities including repos or repo-like products are widely adopted as financing instruments. Such securities transform marketable collaterals with heterogeneous levels of quality, opacity and information friction to immediate funding, and thereby increase funding liquidity and fuel economic growth. Currently, repo financing remains a crucial source of short-term funding for financial institutions, estimated to have an outstanding notional amount of \$12 trillion globally (CGFS, 2017). At the same time, the rise of the market-based financial system has sown the seeds of instability: some short-term borrowing facilities such as asset-backed commercial papers (ABCPs) experienced runs during the Great

[1] According to the Financial Stability Board report on the global shadow banking sector (FSB, 2019), assets under the market-based financial intermediation grew faster than the assets under traditional banks (characterized by the originate-and-hold business models) from 2008 to 2018, and reached 48% of total financial assets at the end of 2018. By the end of 2018, the amount of market-based financial intermediation stood at \$184 trillion compared to \$148 trillion for banks.

Recession. Our theory offers a framework for understanding the potential fragility in the market-based financial system with widespread securitization.

In our model, borrowers value liquidity more than investors, which leads to gains from trade. Borrowers face two commonly observed frictions in raising liquidity. First, they cannot promise to pay back, and thus cannot borrow from investors unless the promise is made credible. To overcome this lack-of-commitment problem and make promises credible, borrowers sell securities backed by the value of a long-lived collateral asset which includes the current period payoff and the endogenous resale price. Second, there is asymmetric information about the quality of the collateral asset which leads to adverse selection. This friction limits the collateral asset's effectiveness in raising liquidity. Under these *two* assumptions on the frictions in the economy, we find that a dynamic price feedback emerges in our model. When the set of available financial instruments is restricted, this dynamic feedback loop leads to multiple equilibria in liquidity provision.

To illustrate how this dynamic price feedback leads to multiplicity, we first consider a baseline case where borrowers are restricted to selling an *equity* claim backed by the collateral asset in the security market to raise liquidity.[2] The quality of the collateral asset (captured by the distribution of its dividend payoff) is either high or low and varies period by period. We focus on the case where the asset quality is independent, identically distributed (i.i.d.) across periods in the main part of the paper, and provide the persistent quality case as an extension. Further, borrowers are privately informed about the current period quality; hence, the security market for the equity claim is subject to adverse selection. At the end of each period, private information is revealed and the collateral asset is traded in an asset market. Therefore, information friction exists exclusively in the security market in this economy. The extent of the information friction in the equity market is related to the asset price level because the equity claim is backed by both the dividend payoff and the asset price. When the asset price is high, the equity claim becomes less informationally sensitive in the sense that the difference between the expected values of low and high quality collateral decreases, and hence the adverse selection problem is milder. Conversely, when the asset price is low, the adverse selection problem is more severe.

There are three possible equilibrium regions in this economy. There is a 'separating' region where adverse selection is severe. In this region, the asset price is depressed. Only borrowers with low-quality assets sell their asset-backed equity claims to obtain funding. Borrowers with high-quality assets choose not to sell any since their equity claims would have been valued at a large discount relative to gains

[2] This restriction is a natural one since equity instruments are available to all economies including those without developed financial markets.

from trade due to severe adverse selection. There is a 'pooling' region where adverse selection is low. In this region, the asset price is booming. Both types raise funding by selling equity claims to acquire high gain from trade and equity claims are priced at a high pooling level. There is also a 'multiplicity' region where adverse selection is intermediate, and both separating and pooling equilibria coexist. That is, in this region prices are self-fulfilling. The price feedback in the model is dynamic because if the equity claim to the asset is traded in a pooling equilibrium in future periods, then the future collateral service of the asset would be large which means that the asset price today would indeed be high. Conversely, if the equity claim to the asset is traded in a separating equilibrium in future periods, then the collateral service of the asset would be low which justifies a lower asset price today. This dynamic feedback loop underlies self-fulfilling asset prices and multiple equilibria in the model.

Next, we introduce the security design. For expositional clarity we make the following modeling choices: 1) in each period, the designer chooses a menu of securities backed by the collateral asset's current period dividend and its resale price to maximize ex ante surplus; 2) securities are traded in dedicated over the counter markets that are informationally segmented; and 3) in each security market investors engage in Bertrand competition. In this environment, we demonstrate that when security design is optimally chosen every period, the dynamic price feedback eliminates the multiplicity and restores the uniqueness in equilibrium.[3] This finding highlights an additional benefit of security design besides the well understood one in the literature – that is, in a static economy optimal security design improves liquidity.[4] Formally, we show that there is a *unique* equilibrium with security design where the optimal design involves a short-term debt tranche that both borrower types sell in a pooling equilibrium, and a residual equity tranche that only the low-type borrower sells in a separating equilibrium. The unique security design equilibrium Pareto-dominates the separating equilibrium and corresponds to the pooling equilibrium in the multiple equilibria region of the equity-only baseline case.

A key economic force in optimal security design is the dynamic feedback between the asset price and the face value of the debt tranche. As the collateral price increases, the debt tranche becomes more valuable and the designer can raise the face value of the debt further. Conversely, as the face value of debt increases, borrowers raise more liquidity by selling debt and hence realizing larger gains from trade,

[3]The assumption that borrowers have the flexibility to adjust the security design at the beginning of each period is important. In practice, security contract terms may not be updated frequently because of administrative costs or simply inattention. Chiu et al. (2022) show that when contract terms are rigid in the sense that the face value may not be updated each period, a run equilibrium through a dynamic price feedback might emerge, and the liquidity of the security market may deteriorate.

[4]For example, Leland and Pyle (1977); Myers and Majluf (1984); Nachman and Noe (1994); DeMarzo and Duffie (1995); Biais and Mariotti (2005) and many others reviewed later.

which leads to higher collateral prices. With security design, agents intertemporally coordinate their beliefs so that the debt tranche is always traded in a pooling equilibrium in each period, leading to a higher price for the collateral asset. A higher asset price, in turn, justifies the debt tranche to be traded in a pooling equilibrium. This dynamic feedback loop removes multiplicity.

In a static setting, it is known since the work of Akerlof (1970) that multiple equilibria may exist in adverse selection models if buyers take prices as given. Wilson (1980) has shown that when buyers are strategic and compete à la Bertrand, there is a unique equilibrium in the static adverse selection environment. We use the equilibrium concept of Wilson (1980) to abstract from the well-known static multiplicity issue and highlight a new dynamic mechanism for multiplicity: the expectation of low prices in the future induces adverse selection in the present, leading to a self-fulfilling low-price equilibrium that survives standard game-theoretic foundations. We further show that a new theoretical result, but in a similar spirit, holds in dynamic settings. Expanding the set of available securities between buyers and sellers eliminates the dynamic multiple equilibria and restores uniqueness.

Moreover, we demonstrate that these insights on price feedback and security design are robust to alternative modeling choices on the markets for securities and the collateral asset. First, we show that our main result carries over to the case of unsegmented security markets where there is information leakage across securities markets. Second, we allow borrowers to issue long-term securities backed by existing long-term securities and show that they can be replicated by the short-term securities (backed by the current period dividend and the collateral asset's resale price) that we study in the main model. Intuitively, the equivalence between these two settings arises because the asset price captures all future gains from trade and is akin to the value function in dynamic programming. Third, instead of Bertrand competition among investors, we assume borrowers and investors interact through an intermediary that maximizes total funding from each security. We show that our baseline multiplicity result holds under this alternative microstructure. Fourth, we allow for noncompetitive pricing of the collateral asset where buyers and sellers bargain over the price of the asset at the end of each period. We show that non-competitive pricing is equivalent to our main model except that borrowers obtain lower gains from trade.

We also extend the model to incorporate persistent private information. When private information is persistent, the asset price is state dependent. Since the state is privately observed by borrowers, persistence introduces an additional source of adverse selection. As a result, when private information is long-lived, the debt tranche is smaller (i.e. there is less leverage) and supports less funding than when private information is short-lived.

In the last part of the paper, we focus on one implementation of the optimal security design: short-term repo contracts. In the repo implementation, there is a representative borrower who values funding liquidity more than the investors. The debt tranche in the optimal design has key characteristics of repo and repo-like contracts: short-term, collateralized debt, typically backed by long-term collateral assets.[5] Asset repurchase arises naturally since the borrower has incentive to purchase back the asset in every period to use it as a collateral backing security issuance in the following period. We provide simple and transparent comparative static results that link repo terms with the model primitives for both types of information frictions, hence deriving new testable implications regarding properties of the repo markets. For example, we find that the repo rate is decreasing and repo haircut is increasing in the degree of adverse selection holding the expected value of the asset's payoff constant.

2 Related Literature

In his seminal work on the lemons market, Akerlof (1970) studies the impact of adverse selection on trade volume and efficiency. There is a long lineage of security design literature with asymmetric information and adverse selection, including Leland and Pyle (1977), Myers and Majluf (1984), Nachman and Noe (1994), DeMarzo and Duffie (1995), DeMarzo and Duffie (1999), Biais and Mariotti (2005).[6] We contribute to this literature by introducing dynamic feedback from asset prices to the security design and illustrating that security design eliminates multiple equilibria.[7]

By studying optimal collateral-backed security design and funding liquidity, our paper is also related to the literature on collateralized lending in monetary economics and macroeconomics starting with the seminal work of Kiyotaki and Moore (1997). Recent studies on macroeconomic implications of financial frictions include Gorton and Ordonez (2014), Kuong (2017), Parlatore (2019), and Miao and Wang (2018). Kurlat (2013) and Bigio (2015) study financial frictions that arise endogenously from adverse selection in a dynamic production economy. Our paper demonstrates that security design in a dynamic

[5] The residual equity tranche can be thought of as a derivative contract traded on the over-the-counter market.

[6] Additionally, Guerrieri, Shimer, and Wright (2010) and Chang (2018) study asset markets with asymmetric information and show that, when buyers post a menu of contracts, screening through probabilistic trading (or market tightness) results in a separating equilibrium.

[7] Our result that both borrower types issue debt that is traded in a pooling equilibrium is reminiscent of Gorton and Pennacchi (1990) and Boot and Thakor (1993). Dang, Gorton, and Holmstrom (2013), Farhi and Tirole (2015) and Yang (2020) incorporate endogenous asymmetric information. The fact that information friction affects the moneyness of an asset has also been studied by Lester, Postlewaite, and Wright (2012) and Li, Rocheteau, and Weill (2012). Security design with heterogenous information is studied in Ellis, Piccione, and Zhang (2017).

adverse selection environment eliminates multiple equilibria, pointing to a potentially socially beneficial role of financial intermediaries.

Our paper is aligned with macrofinance literature where multiple equilibria are dynamic in nature (such as Plantin (2009), Moore (2010; 2013), Chiu and Koeppl (2016), Donaldson and Piacentino (2017), Asriyan, Fuchs, and Green (2019), and Bajaj (2018)). Most closely related to our paper in this literature are Moore (2010; 2013), Chiu and Koeppl (2016), and Asriyan, Fuchs, and Green (2019). They feature dynamic price and liquidity feedback under adverse selection. However, the occurrence of multiple equilibria in these papers crucially depends on the persistence of asset quality. In Asriyan, Fuchs, and Green (2019), for example, the gains from trade are in the style of Duffie, Garleanu, and Pedersen (2007): some agents receive a higher utility from the asset dividend, or produce more output using the asset as input, but they have asymmetric information about the quality of the asset. This structure implies that the severity of the adverse selection problem depends only on what creates gains from trade: dividend. If the asset quality is i.i.d., adverse selection is short lived, future high- and low-quality assets look identical and the future resale concern no longer affects today's adverse selection. In our model, the entire cum-dividend value of the asset creates gains from trade, which generates a new feedback. This insight into dynamic multiplicity is unique and does not depend on persistent asset quality. Furthermore, existing models study dynamic adverse selection in indivisible durable assets, whereas our setup admits divisible financial assets (claims to a stream of future dividend payments), providing a natural setting to study security design. The dynamic multiplicity result in our paper is also similar to the multiplicity result in Bajaj (2018). However, Bajaj (2018) does not study security design, whereas our main theoretical result on uniqueness and our application to repo are about security design.[8]

Finally, the repo implementation of our model is related to theoretical work on repo contracts. Among those, Geanakoplos and Zame (2002), Geanakoplos (2003), Fostel and Geanakoplos (2012), and Simsek (2013) model collateralized borrowing in the general equilibrium context. Gottardi, Maurin, and Monnet (2017) model repo contracts and the repo chain using the competitive approach of Geanakoplos (1997) with an added feature of the nonpecuniary penalty of default. Dang, Gorton, and Holmstrom (2011) model the haircut as the outcome of repo chain, borrower's quality, lender's liquidity need, and collateral value. Bigio and Shi (2020) study a two-period screening model with adverse selection. To attract the high-quality borrowers, lenders offer a repurchase option, a lower rate but have to reduce loan amounts so that default rate is not too high. Therefore, in their model, repos resolve adverse selection inducing

[8]There are also several modeling differences between the two papers. In Bajaj (2018) asset quality is persistent, and the bilateral exchange is modeled as a signaling game.

full participation but introduce cream skimming that can produce worse outcomes than asset sales when adverse selection is mild. Typically in screening models such as Bigio and Shi (2020), the challenge is nonexistence rather than multiple equilibria. We depart from the theoretical literature on repo by modeling the joint determination of collateral asset prices and repo terms, highlighting the unique price feedback mechanism and the role of repo in eliminating price multiplicity.

3 The Model Setup

The economy is set in discrete time and lasts indefinitely. There is a unit of a long-lived asset that pays out a random perishable payoff in every period. There are many infinitely lived potential owners, with deep pockets, identical preferences, and access to the same information, who can potentially own the asset. We refer to a representative owner as agent O. There are also several potential investors who live for a single period and are replaced every period. We refer to them as agent Is.

Gains from Trade. Agent O values inputs provided by the agent Is, which leads to gains from trade in this economy. We denote the value per-unit of the input to agent O by z, and assume that it exceeds the per-unit cost of providing the input by agent Is which is normalized to one.

In a frictionless economy, given that $z > 1$, gains from trade would be potentially unlimited. The key friction that limits gains from trade in our economy is lack of commitment: agent O cannot promise to pay back, and thus cannot borrow from investors unless a credible promise is made. The asset provides a way for agent O to partially overcome the commitment friction because it can be used as collateral to back up payment promises.

Asset Properties and Information Environment. The asset yields a random payoff at the end of period t which we denote as $s_t \in [\underline{s}, \bar{s}]$, where $0 \leq \underline{s} < \bar{s}$. The payoff state, s_t, captures both cash payoff that the asset generates, such as dividend or interest rate payment, and other private benefits that accrue from the asset to agent O, such as a convenience yield and rental income. We assume that s_t is distributed according to probability distribution F_{Q_t} where $Q_t \in \{L, H\}$ denotes the quality of the asset. Quality Q_t is i.i.d. over time and $Q_t = L$ with probability $\lambda \in (0, 1)$.[9]

We denote the density of F_Q by f_Q and its survival function, $1 - F_Q(s)$, by $\widetilde{F}_Q(s)$. We assume both distributions have strictly positive density in their domain $[\underline{s}, \bar{s}]$, and F_H stochastically dominates F_L according to the likelihood ratio, i.e., $f_L(s) / f_H(s)$ is decreasing in $s \in [\underline{s}, \bar{s}]$.

We assume that the use of the collateral asset is, however, limited by an additional friction in the

[9] In Section 8, we study the persistent quality case.

economy: asymmetric information. The quality of the collateral asset is privately observed by the agent O at the beginning of each period, thus introducing an adverse selection problem. The assumption that agent O is better informed of the collateral asset quality can be motivated or microfounded in various ways. For example, borrowers hold collateral assets on their balance sheets which may give them an informational advantage on the quality of these assets.[10] We denote the end-of-period ex-payoff price of the asset by ϕ_t.

Securities. Agent O raises inputs from investors through the sale of asset-backed securities, which are payment promises. Formally, a security $y : [\underline{s}, \bar{s}] \to \mathbb{R}_+$ is a payoff contingent payment contract. Security payment is fulfilled at the end of a period when the state and the price become public information. We assume that securities are monotone:

$$y\left(s\right) \geq y\left(s'\right) \text{ if } s \geq s' \tag{1}$$

for all $s, s' \in [\underline{s}, \bar{s}]$.

A *security design* is a finite set of securities, $\mathcal{J} = \left\{y^1, \cdots, y^J\right\}$, backed by the asset, that is,

$$\sum_{y^j \in \mathcal{J}} y^j\left(s\right) \leq s + \phi_t. \tag{2}$$

for all $s \in [\underline{s}, \bar{s}]$.

Security Markets. The securities are sold in dedicated over-the-counter markets after agent O obtains private information about the asset's quality and before the quality becomes public information. In each security market, several investors make bids à la Bertrand and the seller – the asset owner – decides how much to sell at the highest bid.[11] We denote the price of security j by q_t^j, and the quantity of security j exchanged when the underlying asset quality is Q by $a_{t,Q}^j$.[12] For expositional clarity, we further assume that an investor has access only to one security market, so that trading information is segmented across security markets.[13]

[10]Ownership of the asset often enables owners to observe the cashflows of the asset, or obtain other cashflow-related information (e.g., on governance). One real-world example of this comes from mortgage loans, which are often used as collaterals by loan originators who have better information on their quality. Another example is proprietary investment portfolios of hedge funds that are offered as collateral to obtain financing and increase leverage. There are also historical incidences where some borrowers, especially when hit by unobservable random negative shocks, debased collateral assets, e.g., by reducing the metallic content of coins below their face value. Recently, collateral quality has been subject to questioning because of the possibility that borrowers might pledge it multiple times.

[11]If several investors are tied for the highest bid, agent O equally splits the amount she would like to sell between them.

[12]The price of the security does not depend on the underlying asset quality because investors are not able to distinguish between low and high quality when they make offers for the security, but the quantity exchanged depends on the quality because the owner is privately informed.

[13]In section 7, we show that the segmentation assumption is not necessary for the main results in the paper.

Determination of the Asset Price. Given security design $\mathcal{J}_t$, the ex-ante surplus at time t in this economy is given by:

$$\begin{aligned} V_t = & \lambda \left[\int_{\underline{s}}^{\bar{s}} \left(\sum_{j \in \mathcal{J}_t} a_{t,L}^j \left(z q_t^j - y_t^j(s) \right) + (s + \phi_t) \right) dF_L(s) \right] \\ & + (1-\lambda) \left[\int_{\underline{s}}^{\bar{s}} \left(\sum_{j \in \mathcal{J}_t} a_{t,H}^j \left(z q_t^j - y_t^j(s) \right) + (s + \phi_t) \right) dF_H(s) \right]. \end{aligned} \tag{3}$$

The end-of-period t ex-dividend asset price, ϕ_t is equal to the discounted value of time $t+1$ surplus:

$$\phi_t = \beta V_{t+1} \tag{4}$$

where β is the discount factor, $0 < \beta < 1/z$. The asset price can also be viewed as the continuation value to the owner of retaining the asset at the end of period t. We assume that the asset price is set in a frictionless *competitive centralized* market. In our economy, frictions exist exclusively in the securities market so that we focus purely on the role of security design under dynamic adverse selection.[14]

Security Design Problem. We follow the literature on security design, and assume that the design takes place at the beginning of each period before the arrival of any private information and is flexible since the set of securities can be updated at the beginning of each period. The goal of security design is to choose at the beginning of each period the set of securities that are available for trading in that period to maximize ex-ante surplus V_t taking security prices, q_t^j, and quantities, $a_{t,Q}^j$, as given. We presume an environment with several competing short-lived intermediaries who offer security design services to borrowers. The intermediaries maximize the borrower surplus, which equals the overall surplus since lenders compete Ã -la Bertrand and make zero profits.

Timing. In each period, the security design takes place first. Then, agent O receives private information and trading in the security markets occurs. Once trading in the security markets is completed and gains from trade are obtained, both Q_t and s_t are revealed and asset price is determined. Finally, agent O pays investors who hold the securities, and consumption takes place.[15] Figure 1 graphs this timeline.

We now define the equilibrium concept in our economy.

Definition 1. An equilibrium with security design consists of the asset price ϕ_t, a security design $\mathcal{J}_t = \left\{ y_t^1, \cdots, y_t^J \right\}$, security prices q_t^j and quantities $\left\{ a_{t,L}^j, a_{t,H}^j \right\}$ for all securities such that:

[14] In section 7, we allow for a noncompetitive mechanism where the asset price is set via Nash bargaining.

[15] The dynamic framework is borrowed from Lagos and Wright (2005).

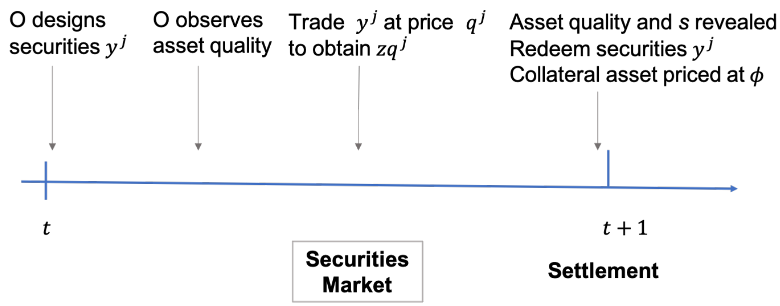

Figure 1: Timeline

1. The price of security j, q_t^j, is determined through Bertrand competition in each security market, and thus q_t^j is equal to the expected value of a unit of the security given $a_{t,Q}^j$:

$$q_t^j = \lambda a_{t,L}^j E_L y_t^j(s) + (1-\lambda) a_{t,H}^j E_H y_t^j(s). \tag{5}$$

2. Quantities sold by each type must be optimal given the price, i.e., for each $Q \in \{L, H\}$,

$$a_{t,Q}^j \in \arg\max_{a\in[0,1]} a\left(zq_t^j - E_Q y_t^j(s)\right). \tag{6}$$

3. Asset price ϕ_t satisfies (4).

4. Security design $\mathcal{J}_t$ satisfies constraints (1) and (2) and maximizes (3) among all security designs satisfying (1) and (2) where security prices and quantities are given by (5) and (6).

For the reminder of this paper, we study *stationary equilibria* and hence remove the time subscripts.

4 Equilibrium in Security Markets

The value of securities affects the optimal security design. We begin the analysis by describing the equilibrium in the market for an arbitrary security y. We assume that the expected payoff of the security when issued by the high type is weakly more than that issued by the low type, i.e., $E_L y(s) \leq E_H y(s)$.[16] We define the degree of information insensitivity as the ratio of the expected value of the security under the low versus the high distribution, i.e., $E_L y(s)/E_H y(s)$. As this ratio increases, the expected values of the security under the low versus high distribution become closer, and the adverse selection problem becomes less severe.

[16]This assumption is automatically satisfied for monotone securities.

Since our focus is not on multiple equilibria in the static setting, following Wilson (1980), we assume that investors are strategic and compete à la Bertrand, ensuring that the equilibrium in each security market is generically unique. That is, in the market for security y, investors simultaneously make price offers taking into account which types of borrowers would sell the security at that price. Agent O observes these offers, and decides how much of the security to allocate to each investor.[17] Due to Bertrand competition, investors make zero surplus in expectation, and the equilibrium price of the security, q, is given by (5). The quantity sold by each type of agent O, a_Q, is optimal for that type and satisfies (6). The next proposition characterizes the equilibrium in the security market.

Proposition 1. *If $E_L y(s)/E_H y(s) > \zeta \equiv 1 - (z-1)/(\lambda z)$, in the market for security y the price of the security is $q = \lambda E_L y(s) + (1-\lambda) E_H y(s)$ and $a_L = a_H = 1$. If $E_L y(s)/E_H y(s) < \zeta$, the price of the security is $q = E_L y(s)$ and $a_L = 1$ and $a_H = 0$.*

Proposition 1 shows that when $E_L y(s)/E_H y(s)$ is above the threshold ζ, the adverse selection problem is not too severe, and both types sell the security. In this case, the security price is the pooling price $q = \lambda E_L y(s) + (1-\lambda) E_H y(s)$. When $E_L y(s)/E_H y(s)$ is below the threshold, the adverse selection problem is severe, and only the low type sells the security. In this case, the security price is the separating price $q = E_L y(s)$. A security traded in a pooling equilibrium commands a higher price and generates more liquidity for the borrower than the one traded at a lower separating equilibrium price. When $E_L y/E_H y = \zeta$, both pooling and separating (and even semi-separating) equilibria are possible. To simplify exposition in this knife edge case, we select the pooling equilibrium.

The above proposition also indicates that in addition to the parameters that characterize quality heterogeneity, the gains from trade parameter, z, is also an important determinant of adverse selection: a lower z leads to a higher ζ. Even if there is very little asymmetric information about the quality of the security i.e., when $E_L y(s)/E_H y(s)$ is slightly below 1, as z approaches 1 (so that ζ is close 1), the security will be sold in a separating equilibrium. In other words, when gains from trade are low, even a slight amount of asymmetric information results in adverse selection problem.

[17] In this formulation agent O has all the bargaining power, but this is not critical for any of our results. In section 7, we allow for a noncompetitive mechanism where the asset price is set via Nash bargaining.

5 The Baseline: Multiple Equilibria of the Dynamic Lemons Market

In this section, we consider a baseline case where the borrower is restricted to issuing only the equity claim, or a passthrough security, to the collateral asset in the security market. We demonstrate that this economy is fragile and exhibits dynamic multiplicity in prices. Specifically, we show that there might be multiple equilibria in the security market justified by different asset prices. The different asset prices are themselves justified by the different equilibria in the security market.

For this baseline case, we use the notion of equilibrium in Definition 1 with the restriction that the equity claim to the asset is the only available security. Security design becomes trivial since there is only a single feasible security. By Proposition 1 the price of the equity claim to the asset in the security market is given by $q^P = \lambda E_L s + (1-\lambda) E_H s + \phi$ if $(E_L s + \phi)/(E_H s + \phi) \geq \zeta$ and $q^S = E_L s + \phi$ otherwise. Using (4), we obtain the price of the collateral asset in the asset market as:

$$\phi = \begin{cases} \beta z q^P, & \text{if } \frac{E_L s+\phi}{E_H s+\phi} \geq \zeta, \\ \beta \left[z\lambda q^S + (1-\lambda)\left(E_H s + \phi\right)\right], & \text{if } \frac{E_L s+\phi}{E_H s+\phi} < \zeta. \end{cases} \tag{7}$$

Note that in a stationary equilibrium, the equity claim backed by the collateral asset is either always traded in a pooling equilibrium, or always traded in a separating equilibrium.

5.1 Pooling Equilibrium

Plugging q^P into (7) shows that a pooling equilibrium, in which both types of agent O sell the equity claim in the security market, exists if and only if

$$\frac{E_L s + \phi^P}{E_H s + \phi^P} \geq \zeta, \tag{8}$$

where the asset price in the pooling equilibrium is given by:

$$\phi^P = \beta z \left(\lambda E_L s + (1-\lambda) E_H s + \phi^P\right).$$

Solving for the pooling price we obtain:

$$\phi^P = \frac{\beta z \left(\lambda E_L s + (1-\lambda) E_H s\right)}{1-\beta z}. \tag{9}$$

Plugging (9) into (8) shows that a pooling equilibrium exists if and only if $E_L s / E_H s \geq \kappa_P$, where

$$\kappa_P = \frac{\zeta - \beta z \left(1 - (1-\zeta)\lambda\right)}{1 - \beta z \left(1 - (1-\zeta)\lambda\right)}.$$

5.2 Separating Equilibrium

A separating equilibrium, in which only the low type of agent O sells the equity claim in the security market, exists if and only if:

$$\frac{E_L s + \phi^S}{E_H s + \phi^S} < \zeta, \tag{10}$$

where the asset price in the separating equilibrium is given by:

$$\phi^S = \beta\left(z\lambda E_L s + (1-\lambda) E_H s + (z\lambda + (1-\lambda))\phi^S\right),$$

Solving for the separating price, we obtain:

$$\phi^S = \beta\frac{z\lambda E_L s + (1-\lambda) E_H s}{1-\beta-\beta\lambda(z-1)}. \tag{11}$$

Plugging (11) into (10) indicates that a separating equilibrium exists if and only if $E_L s/E_H s < \kappa_S$, where:

$$\kappa_S = \frac{\zeta - \beta(1-(1-\zeta z)\lambda)}{1-\beta(1-(1-\zeta z)\lambda)}.$$

5.3 Properties of Equilibria and Multiplicity

The ratio $E_L s/E_H s$ captures the degree of adverse selection in the equity market of the baseline case. As this ratio increases, the expected payoff with respect to the two distributions becomes closer, and adverse selection is ameliorated. It is easy to check that $\kappa_P < \kappa_S$. Hence, there is always an intermediate degree of adverse selection where multiple equilibria exist. We present this result in the following proposition.

Proposition 2. *(i) If $E_L s/E_H s \geq \kappa_S$, then there is a unique equilibrium in which the equity claim of collateral asset is sold in a pooling equilibrium in the security market, and the pooling price is given by (9).*

(ii) If $\kappa_P > E_L s/E_H s$, then there is a unique equilibrium in which the equity claim of the collateral asset is sold in a separating equilibrium in the security market, and the separating price is given by (11).

(iii) If

$$\kappa_S > \frac{E_L s}{E_H s} \geq \kappa_P, \tag{12}$$

then both the pooling equilibrium described in (i), and the separating equilibrium described in (ii) exist.

The intuition for the multiple equilibria result in this proposition is as follows. When the asset price is high, the degree of information insensitivity of equity, $\left(E_L s + \phi^P\right)/\left(E_H s + \phi^P\right)$, is above the threshold ζ. Hence, the adverse selection problem is mild, and the high-type agent O is willing to pool with the low type and issue equity in the security market. In turn, if agents anticipate the equity claims of the asset to be traded in a pooling equilibrium in future periods, the liquidity service of the asset is large; hence, the asset price today is high. Conversely, when the asset price is low, the degree of information insensitivity of the equity, $\left(E_L s + \phi^S\right) / \left(E_H s + \phi^S\right)$, is below the threshold ζ. Therefore, the adverse selection problem is severe and the high type agent O retains the asset and chooses not to issue equity in the security market. In turn, if agents anticipate the equity claim of the asset traded in a separating equilibrium in future periods, the liquidity service of the asset is limited; thus, the asset price today is low. As a result, asset prices are self-fulfilling in this economy.

In the baseline case, multiple equilibria exist even though the asset quality is i.i.d. This fact indicates that the sources of multiple equilibria in our setting are distinct from those in the existing literature. In the static setting, multiple equilibria exist under perfect competition as in Akerlof (1970). If prices are low, only the equity claims of the low-quality assets are sold, which justifies low prices; if prices are high, the equity claims of the higher-quality assets are also sold, which in turn justifies higher prices. For some parameter values both equilibria exist. Wilson (1980) has shown that when buyers are strategic and compete à la Bertrand, there is a unique equilibrium in the static adverse selection environment. The reason is that uninformed buyers do not take prices as given, they recognize the link between price and quality, and only the highest zero-profit price survives as a Nash equilibrium. However, this logic does not extend to dynamic settings. The expectation of low prices in the future could induce adverse selection in the present thus lead to a self-fulfilling low-price equilibrium that survives standard game-theoretic foundations.

In the next section, we show that increasing the flexibility of security design by removing the restriction on the set of available securities, restores the unique equilibrium in the economy.

6 The Main Model: Optimal Security Design

In this section, we solve the equilibrium described in Definition 1 with security design, and show that the equilibrium is unique. Hence, optimal security design eliminates multiple equilibria that arise when agents are restricted to trading only the equity claim of the underlying asset.

6.1 Unique Equilibrium with Optimal Security Design

The next proposition characterizes the optimal security design, and shows that it involves at most two securities: One security, $y_D(s)$, which is traded in a pooling equilibrium, is a debt contract; the other security, $y_E(s)$, which is traded in a separating equilibrium, is the residual equity tranche. That is, both high and low quality borrowers sell one unit of the debt contract, only low quality borrowers sell one unit of the equity contract and high quality borrowers retain the equity contract.

Proposition 3. *The optimal security design consists of two security tranches. One tranche is a debt contract given by:*

$$y_D(s) = \min(s + \phi, D), \tag{13}$$

for some $D \in (\underline{s}+\phi, \bar{s}+\phi]$. The residual tranche is an equity contract given by $y_E(s) = \max(0, s + \phi - D)$. Moreover, the debt contract is traded in a unique pooling equilibrium, the equity contract is traded in a unique separating equilibrium and, D is unique for a given ϕ.

The amount D is the face value of the debt contract, and it is pinned down by the high-quality asset owner's participation constraint. The face value of debt always exceeds $\underline{s} + \phi$, and incorporates the lower bound of asset payoff (e.g., dividend or interest payment) and the asset price since this amount is free from adverse selection.

Using Proposition 3 and letting $d \equiv D - \phi$, we simplify the statement of equilibrium given in Definition 1. With this notation, we write the prices of the debt and equity tranches, q_D and q_E, as:

$$q_D = \lambda \left(\phi + E_L s - \int_d^{\bar{s}} \widetilde{F}_L(s) ds \right) + (1 - \lambda) \left(\phi + E_H s - \int_d^{\bar{s}} \widetilde{F}_H(s) ds \right), \tag{14}$$

$$q_E = \int_d^{\bar{s}} \widetilde{F}_L(s) ds. \tag{15}$$

Both types of borrowers sell the debt tranche, but only the low type sells the equity tranche. As a result, the expected amount raised by selling the securities equals $q_D + \lambda q_E$. From (4), the asset price can be written as:

$$\phi = \beta \left[z q_D + z \lambda q_E + (1 - \lambda) \left(\int_d^{\bar{s}} \widetilde{F}_H(s) ds \right) \right]. \tag{16}$$

Solving for equilibrium then comprises solving the designer's optimization problem to find the optimal threshold $d \in (\underline{s}, \bar{s}]$ given the prices of debt and equity tranches q_D and q_E, and the asset price ϕ.

We state the main theorem of this paper as follows.

Theorem 1. *There is a unique equilibrium with security design. If*

$$E_L s / E_H s < \kappa_P, \tag{17}$$

then the debt threshold $d \in (\underline{s}, \bar{s})$ and the asset price ϕ are solutions to the participation constraint and the Euler equation that are given by:

$$\phi = \frac{z}{z-1}\lambda \int_{\underline{s}}^{d} \left[\widetilde{F}_H(s) - \widetilde{F}_L(s)\right] ds - \int_{\underline{s}}^{d} \widetilde{F}_H(s) ds - \underline{s}, \tag{18}$$

$$\phi = \frac{\beta}{1-\beta z} \left\{ z\left[\lambda E_L s + (1-\lambda) E_H s\right] - (1-\lambda)(z-1) \int_{d}^{\bar{s}} \widetilde{F}_H(s) ds \right\}. \tag{19}$$

If $E_L s / E_H s \geq \kappa_P$, then $d = \bar{s}$ and $\phi = \frac{\beta z}{1-\beta z}\left[\lambda E_L s + (1-\lambda) E_H s\right]$.

In the former case, the equilibrium with security design strictly Pareto dominates the (unique) separating equilibrium in the baseline case. In the latter case, security design uniquely selects the pooling equilibrium. Thus, it strictly Pareto dominates the separating equilibrium in the baseline case when there is one, and it replicates the pooling equilibrium otherwise.

The region given by (17) is the same region identified in Proposition 2 where a unique separating equilibrium exists in the equity-only baseline case. Hence, security design improves liquidity when there is a unique separating equilibrium in the equity-only baseline case.

We note that for the self-fulfilling multiple equilibria result in Proposition 2 and the uniqueness under optimal security design result in Theorem 1 to hold, we only need the following two assumptions: lack of commitment from borrowers and asymmetric information about the quality of the (only pledgeable) collateral asset. We demonstrate later in this paper that the main results are robust to alternative securities and asset market microstructures. The modeling choices we have made in the main model, such as segmented securities markets, competitive asset markets, Bertrand competition in securities markets, as well as maturity structures, are mainly for expositional clarity.

Figure 2 illustrates the feedback loop between the asset price, which depends on the future value of the collateral, and the current face value of the debt contract, which is the underlying mechanism in Theorem 1. As the face value of the debt tranche, $D = \phi + d$, increases, agent O obtains more liquidity, and gains from trade increase because the marginal value of liquidity for agent O exceeds the marginal cost of providing liquidity by the investors. The feedback loop involves intertemporal coordination since the increase in gains from trade in future periods leads to an increase in ϕ. A higher asset price is incorporated into the face value of debt, alleviating the adverse selection problem and pushing the face value even higher.

Next, to provide intuition for the result in Theorem 1, we graphically construct the optimal security design equilibrium. For any d, let $\phi(d)$ be the asset price in the asset market satisfying (19). Similarly, for any ϕ, let $d(\phi)$ be the debt threshold satisfying 18. We graph the former with a solid line and the

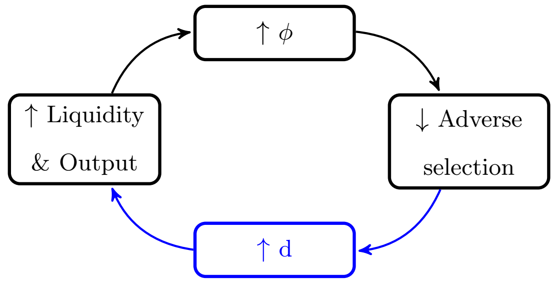

Figure 2: Asset Price ϕ and Debt Face Value $\phi + d$

latter with a dashed-dotted line in Figures 3 and 4. An intersection of these two lines is a solution to (18) and (19) and constitutes an equilibrium.

The Euler equation (19) shows that if agents coordinate on a higher debt threshold tomorrow, the asset price today will be higher, since ϕ is increasing in d.[18] The function $\phi(d)$ has a few noteworthy aspects. Let $\underline{\phi} = \phi(\underline{s})$ and $\phi^P = \phi(\bar{s})$. From (11) we observe that ϕ^S is the asset price when only the low type sells the asset and the high type retains both the resale price and the current period payoff. In contrast, the asset price calculation in (19) takes into account that both types of borrowers sell the debt claim backed by the future resale price as part of the collateral. As a result, $\underline{\phi} > \phi^S$. On the other hand, $\phi(\bar{s})$ is the same as the pooling price ϕ^P. To see this, we observe that the ϕ^P calculation takes into account that both types use the resale price and the entire current period payoff of the asset as collateral, which is equivalent to setting the face value of the debt contract to $D = \phi^P + \bar{s}$.

Next, we consider the designer's choice of debt threshold, $d(\phi)$, which is depicted by the dashed-dotted line in Figure 3 for the case where $E_L s / E_H s < \kappa_P$.[19] The optimal security design chooses d to be as large as possible making sure that the debt tranche is traded in a pooling equilibrium. We discussed in the previous paragraph that as d increases, ϕ increases, which is depicted by the solid line in Figure 3. This relaxes the high type's participation constraint. However, as the debt tranche incorporates more of the high payoff states, eventually the high type's participation constraint begins to tighten because, by the likelihood ratio dominance, the likelihood of the high payoff states according to the high type relative to the low type keeps increasing, and the adverse selection problem worsens. If d is too high, the high type, who values those states much more than the low type, might prefer to retain the debt tranche rather than pool with the low type. The optimal security design pushes d to the

[18]Note that ϕ is strictly increasing for $d \in [\underline{s}, \bar{s})$, $\partial\phi/\partial d$ is decreasing and is zero at $d = \bar{s}$.

[19]This is the left boundary of the multiple equilibria region in 12. In this region, without security design, adverse selection leads to a unique separating equilibrium.

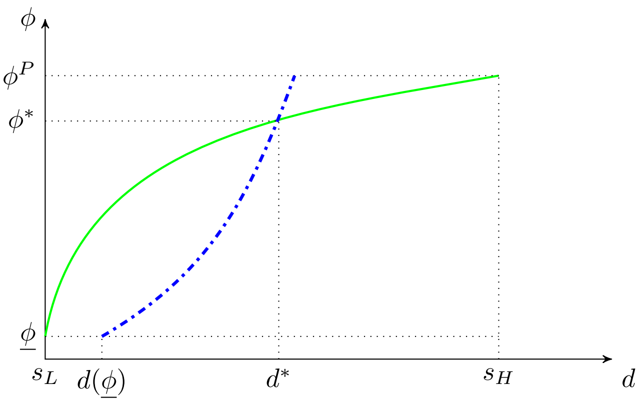

Figure 3: $\phi(d)$ and $d(\phi)$ when $E_L s/E_H s < \kappa_P$

unique point where the high type is indifferent between selling or retaining the debt. Crucially, optimal security design solves the coordination problem that we observed in the baseline case where lenders face strategic uncertainty about the high type's participation in the security market. Optimal security design eliminates this uncertainty by ensuring that both types participate in trading the debt tranche.

Figure 3 illustrates that regardless of how low the asset price is, as long as tranching is feasible, optimal security design involves a debt tranche that incorporates some of the current period payoff. That is, $d\left(\underline{\phi}\right) > \underline{s}$. In the region depicted in Figure 3, adverse selection is severe, and even when the asset price is as high as possible, the high type prefers to retain the equity tranche. That is, $d\left(\phi^P\right) < \bar{s}$.

Using these two curves, $\phi(d)$ and $d(\phi)$, we can find the equilibrium values (d^*, ϕ^*). The equilibrium is where the two curves intersect, i.e., when $\phi^* = \phi(d^*)$ and $d^* = d(\phi^*)$. As Figure 3 shows, when $E_L s/E_H s < \kappa_P$, the unique equilibrium debt threshold is $d^* \in (\underline{s}, \bar{s})$. This explains the optimal security design equilibrium and its difference relative to the baseline case in the first scenario.

We next consider the scenario when $E_L s/E_H s > \kappa_P$ in Figure 4. In this case, adverse selection is less severe and the $d(\phi)$ function is shifted to the right as the same asset price can sustain a higher face value where the debt tranche is traded in a pooling equilibrium. When the asset price is above a threshold denoted by $\widehat{\phi}$, optimal security design incorporates all payoff states $\bar{s}$ to the face value of debt, which is captured by the vertical part of the $d(\phi)$ function. This vertical portion of $(d(\phi))$ is a special feature of debt contracts: the debt threshold cannot exceed the maximum payoff that the collateral asset can yield. The two curves intersect only at the upper right corner, $\left(\bar{s}, \overline{\phi}\right)$. As a result, there is a unique equilibrium for the security design problem and it involves setting the debt threshold $d^* = \bar{s}$. That is, the optimal

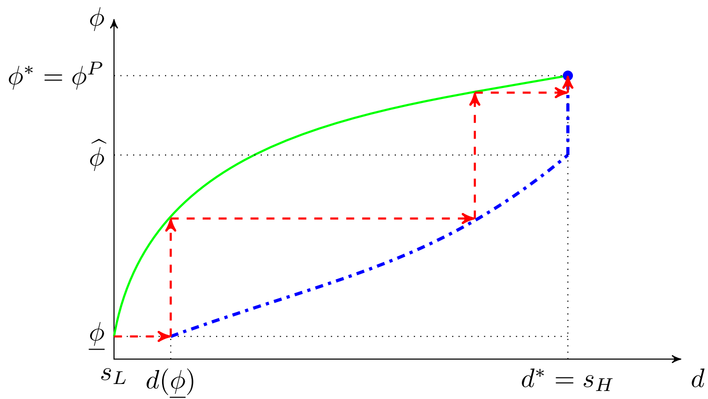

Figure 4: $\phi(d)$ and $d(\phi)$ when $E_L s/E_H s > \kappa_P$

security is a pass-through security, which means that the optimal security's payoff is mapped one-to-one from the asset's cashflow at the realization date, equivalent to an equity contract.

The scenario depicted in Figure 4 may seem surprising since, as we illustrated in Section 5, without the possibility of security design, there is a coordination problem leading to multiple equilibria in part of this region. Security design solves this coordination problem, and we obtain a unique equilibrium in which agent O sells the entire "pass-through" debt tranche in a pooling equilibrium. Intuitively, without security design, the high type decides among only two options: whether to use the resale price and the current period payoff of the asset as collateral versus retaining both parts. The outcome depends on the asset price. In the good equilibrium $\phi = \phi^P$ and the high type sells the asset. In the bad equilibrium, $\phi = \phi^S$ and the high type retains the asset. The bad equilibrium cannot survive with security design because even if the asset price was ϕ^S, the optimal security design would be able to improve this separating equilibrium by creating a debt tranche with face value ϕ^S, which in turn would increase the asset price above ϕ^S. Both graphs in Figures 3 and 4 in fact show that the equilibrium asset price in the optimal security design equilibrium is no less than $\underline{\phi} = \phi(\underline{s}) > \phi^S$ (since the face value of the debt tranche is never below $\underline{\phi} + \underline{s}$). Given the increase in the asset price to $\underline{\phi}$ from ϕ^S, the high type's participation constraint is relaxed, which leads to the optimal security design to incorporate more of the current period payoff into the debt tranche (that is, $d > \underline{s}$). A higher d will increase the asset price ϕ and so on, triggering the dynamic price feedback loop. This unravelling process is illustrated in Figure 4 with the dashed arrows. As the graph in Figure 4 shows, when the asset price is $\underline{\phi}$, the face value of the debt rises to $\underline{\phi} + d\left(\underline{\phi}\right)$. When the face value of the debt increases to $\underline{\phi} + d\left(\underline{\phi}\right)$, the asset price further increases. The process ends when the price rises to ϕ^P.

7 Robustness: Alternative Modeling Choices

We now demonstrate the robustness of our results derived from the main model by allowing for alternative modeling choices. For expositional clarity, we only provide a summary and intuitions in this section. The formal results are presented in the online Appendix B.

Unsegmented Security Markets. We first change the segmentation assumption on security markets by assuming security markets are unsegmented, that is, we allow lenders to make inferences about the type of the borrower from their trades across markets. We show in Appendix B.1. that our main result – there is a unique equilibrium with security design – is robust to this modification.

With unsegmented markets, the security designer chooses at most two securities. When high and low–type borrowers trade different securities, the design is separating. When both types trade the same security, the design is pooling. In the separating case, the designer chooses a debt tranche and pass-through equity. The high type trades only the debt tranche (and retains the residual equity) and the low type trades the pass-through equity. Because security markets are unsegmented, in the separating case, lenders learn the borrower's type. We show that in this case, the only constraint that binds is the low type's incentive compatibility (IC) constraint which ensures that the low type does not mimic the high type by selling debt instead of pass-through equity. In the separating case, this constraint pins down the debt threshold. In the pooling case, the designer chooses a single debt tranche. Both types trade the debt and retain the residual equity. We show that in this case, the only constraint that binds is the high type borrower's participation constraint which ensures that the high type has the incentive to sell debt instead of retain it.[20] In the pooling case, this constraint pins down the debt threshold. We show that overall, there is still a unique equilibrium in which either the design is separating and the equilibrium asset price is low, or the design is pooling and the asset price is high. An immediate corollary of the analysis is that the designer obtains a higher payoff with segmented compared with unsegmented markets since the low type's IC constraint is not needed in the former case.

Long-term Securities. In our main model, the borrower sells a short-term security backed by the current period dividend and the resale price of the long-lived asset. We demonstrate that the restriction of security design to short-term securities is not as restrictive as it might seem. To do so, we introduce long-term securities that specify payments from the borrower to investors in every period and state.

[20]This constraint is the same as the high type's participation constraint in our main model.

We drop the assumption that investors are short-lived. Instead, we assume that an investor who buys a long-term security becomes a borrower in the next period and raises inputs by designing and selling another long-term security backed by the existing long-term security that she owns.

We compare the setting with long-term securities with the one we consider in our main model. In Appendix B.2, we first show that these two environments are equivalent – in the sense that the amount of inputs raised in the securities market and borrower's continuation value are the same – under symmetric information. This result is intuitively similar to the principle of optimality in dynamic programming: the asset price captures all future gains from trade and is akin to the value function which captures the value of the dynamic program under future optimal behavior. We then illustrate how the equivalence result also extends to the asymmetric information where quality is i.i.d. when the borrower issues securities that are long-term debt-like and the residual equity tranche.

Securities Market Microstructure. In the main model, we stay close to the standard lemons market Ã la Akerlof which is the simplest model of a lemons market and provide closed-form solutions. To show that our results are robust to a different security market microstructure, we solve a model where the borrower and investors trade securities through an intermediary in Appendix B.3. We assume that the intermediary's goal is to maximize the expected amount of inputs raised in the security market while making sure that lenders break even. An alternative interpretation of this setting is that the borrower signal her type through the quantity that she trades (or equivalently probability of trade) and we select the most efficient equilibrium. The latter interpretation is closely related to the undefeated equilibrium concept of Mailath, Okuno-Fujiwara, and Postlewaite (1993) invoked in Bajaj (2018). We show that securities are still traded either in a pooling or a separating equilibrium. However, unlike in the main model, the high type is able to sell a fraction of the security in the separating case. Despite this difference, there is still a discontinuous drop in the amount of inputs the borrower can raise when equilibrium switches from pooling to separating. We show that, as in our main model, when the borrower is restricted to issuing only the equity claim to the collateral asset, the economy exhibits dynamic multiplicity.

Nash Bargaining for Asset Resale Price. In the main model, we assume that at the end of each period, there is a competitive market where the borrower buys back the asset from the investor. Suppose instead, the two parties bargain over the resale price of the asset via Nash bargaining where $\theta \in (0, 1]$ is the bargaining power of the borrower. We show in Appendix B.4 that this alternative model is equivalent to the main model where the gains from trade parameter z in the asset price is replaced with $\hat{z} = 1 - \theta + \theta z$. This result is intuitive since only proportion θ of the gains from trade is captured by the borrower and hence, reflected in the asset price.

8 Persistent Asset Quality

In this section, we discuss how introducing persistence in asset quality affects the feedback loop and adverse selection problem in our model. To capture persistence, we assume that asset quality $Q_t \in \{L, H\}$ follows a Markov process where $Q_t = L$ with probability $\lambda_{Q_{t-1}} \in (0,1)$, where $\lambda_L \geq \lambda_H$. The unconditional probability of $Q_t = L$ in the steady state is denoted by λ, where $\lambda \equiv \lambda_H / (1 - \lambda_L + \lambda_H)$. Persistence in quality increases when $\lambda_L - \lambda_H$ increases holding $\lambda \in (0,1)$ constant. We assume that financial markets are segmented across time so that when the period is over, agents in the following period cannot access the past trading or payoff information (including the security designer). When quality is i.i.d., i.e., $\lambda_L = \lambda_H$, this assumption is innocuous because past quality does not provide any information about the future. It allows us to abstract away from the issue of signaling and reputation and focus instead on the dynamic coordination role of security design.[21]

In the persistent quality setup, since Q is observed at the end of the period, the ex-dividend asset price depends on the quality realization. Hence, we need to change a few notations accordingly in the security design problem of the i.i.d case. First, we denote the asset price by $\phi_Q : \{L, H\} \to \mathbb{R}$. Note that, in addition to the current period payoff, now lenders and borrowers are asymmetrically informed about the future asset price. As we show below, this additional source of information asymmetry makes the adverse selection problem more severe in the persistent case.

Second, the feasible securities are now assumed monotone in the total payoff. That is,

$$y(s, \phi) \geq y(s', \phi') \text{ if } s + \phi \geq s' + \phi'. \tag{20}$$

Intuitively, since security payments are fulfilled at the end of a period when the state and the price become public information, securities now depend explicitly on the end-of-period price, which depends on the quality realization.

Therefore, in this setting, the asset price at the end of period t, $\phi_{t,Q}$ for $Q \in \{L, H\}$, is expressed as its discounted value given time t quality and time $t+1$ security design $\mathcal{J}_{t+1}$:

[21]See Chari, Shourideh, and Zetlin-Jones (2014) on the reputation effect under adverse selection.

$$\phi_{t,Q} = \beta \left\{ \lambda_Q \left[\int_{\underline{s}}^{\bar{s}} \left(\sum_{j \in \mathcal{J}_{t+1}} a^j_{t+1,L} \left(zq^j_{t+1} - y^j_{t+1}(s, \phi_{t+1,L}) \right) + (s + \phi_{t+1,L}) \right) dF_L(s) \right] \right. \tag{21}$$
$$\left. + (1-\lambda_Q) \left[\int_{\underline{s}}^{\bar{s}} \left(\sum_{j \in \mathcal{J}_{t+1}} a^j_{t+1,H} \left(zq^j_{t+1} - y^j_{t+1}(s, \phi_{t+1,H}) \right) + (s + \phi_{t+1,H}) \right) dF_H(s) \right] \right\},$$

Finally, in the beginning of each period t, the security design $\mathcal{J}_t$ takes the prices, q^j_t, and quantities, $a^j_{t,Q}$, as given to maximize:

$$V_t = \lambda \left[\int_{\underline{s}}^{\bar{s}} \left(\sum_{j \in \mathcal{J}_t} a^j_{t,L} \left(zq^j_t - y^j_t(s, \phi_{t,L}) \right) + (s + \phi_{t,L}) \right) dF_L(s) \right] \tag{22}$$
$$+ (1-\lambda) \left[\int_{\underline{s}}^{\bar{s}} \left(\sum_{j \in \mathcal{J}_t} a^j_{t,H} \left(zq^j_t - y^j_t(s, \phi_{t,H}) \right) + (s + \phi_{t,H}) \right) dF_H(s) \right].$$

In the Online Appendix, we show that the baseline multiplicity result when the borrower is only allowed to issue equity extends to the case of persistent quality, and the multiplicity region expands when persistence in asset quality increases. The next proposition extends Proposition 3 to the persistent quality case.

Proposition 4. *Suppose that either f_L or f_H is log-concave. The optimal security design consists of two security tranches. The tranche traded in a pooling equilibrium is a debt contract given by:*

$$y_D(s, \phi_Q) = \min(s + \phi_Q, D), \tag{23}$$

for some $D \in (\underline{s}+\phi_L, \bar{s}+\phi_H]$. The residual tranche is an equity contract traded in a separating equilibrium and is given by $y_E(s, \phi_Q) = \max(0, s + \phi_Q - D)$. Moreover, D is unique for given ϕ_L and ϕ_H.

We present the proofs of all the results in this section in the Online Appendix.

With persistence, the face value of debt always incorporates exceeds $\underline{s} + \phi_L$ since this amount is free from adverse selection but may be less than $\underline{s} + \phi_H$ because the asset price itself is now subject to adverse selection. As the persistence in asset quality vanishes and λ_L and λ_H get closer, and the resale prices ϕ_L and ϕ_H approach each other. In the limit we obtain the i.i.d. case where $\phi_L = \phi_H = \phi$, and the face value of debt always exceeds $\underline{s} + \phi$, incorporating all of the resale price.

Let $d \equiv D - \phi_L$ and $\Delta\phi \equiv \phi_H - \phi_L$.[22] With this notation, we write the prices of the debt and equity

[22]When the asset quality is low, d is the threshold up to which the current period's payoff is incorporated into the debt tranche. When the asset quality is high and $d - \Delta\phi < 0$, the debt tranche incorporates a fraction of the asset price ϕ_H, namely $D < \phi_H$, and none of the current payoff. When the asset quality is high and $d - \Delta\phi > 0$, the debt tranche incorporates the asset price ϕ_H and the current payoff up to the threshold $d - \Delta\phi$.

tranches, q_D and q_E, as:

$$q_D = \lambda \left(\phi_L + E_L s - \int_d^{\bar{s}} \widetilde{F}_L(s) ds \right) + (1-\lambda) \left(\phi_H + E_H s - \int_{d-\Delta\phi}^{\bar{s}} \widetilde{F}_H(s) ds \right), \tag{24}$$

$$q_E = \int_d^{\bar{s}} \widetilde{F}_L(s) ds. \tag{25}$$

With persistence, the expected amount of inputs raised by selling the securities depends on the state Q and equals $q_D + \lambda_Q q_E$. From (4), we write the asset prices as:

$$\phi_Q = \beta \left[z q_D + z \lambda_Q q_E + (1-\lambda_Q) \left(\int_{d-\Delta\phi}^{\bar{s}} \widetilde{F}_H(s) ds \right) \right], \; Q \in \{L, H\}. \tag{26}$$

Solving for equilibrium then comprises solving the designer's optimization problem to find the optimal threshold $d \in [\underline{s}, \bar{s} + \Delta\phi]$ given the prices of debt and equity tranches q_D and q_E, and asset prices ϕ_L and ϕ_H.

For the next theorem, we strengthen the standard hazard rate dominance condition, and assume that F_H and F_L satisfy:

$$\left(f_H(s) / \widetilde{F}_H(s) \right) / \left(f_L(s) / \widetilde{F}_L(s) \right) \leq [1 - \beta(\lambda_L - \lambda_H)]^2. \tag{27}$$

In the i.i.d. case, where $\lambda_L = \lambda_H$, (27) is the standard hazard rate dominance condition, which automatically follows from the likelihood ratio dominance assumption.[23]

The generalization of Theorem 1 to the case of persistent quality is as follows.

Theorem 2. *Suppose that either f_L or f_H is log-concave, and (27) holds, then there is a unique equilibrium with security design. If*

$$E_L s / E_H s < \kappa_P, \tag{28}$$

then $d \in (\underline{s}, \bar{s} + \Delta\phi)$, otherwise, that is, if $E_L s / E_H s \geq \kappa_P$, then *$d = \bar{s} + \Delta\phi$. In the former case, the equilibrium with security design strictly Pareto dominates the (unique) separating equilibrium in the baseline case. In the latter case, security design uniquely selects the pooling equilibrium. It thus strictly Pareto dominates the separating equilibrium in the baseline case when there is one and it replicates the pooling equilibrium otherwise.*

The next proposition shows that an increasing persistence in asset quality leads to a lower debt threshold.

[23] This is because when F_H stochastically dominates F_L according to the likelihood ratio, i.e., $f_L(s)/f_H(s)$ is decreasing in s, then F_H stochastically dominates F_L according to hazard rate, that is,

$$\left(f_H(s) / \widetilde{F}_H(s) \right) / \left(f_L(s) / \widetilde{F}_L(s) \right) \leq 1.$$

More generally, (27) is stronger than the hazard rate dominance condition and is not implied by likelihood dominance.

Proposition 5. *Fix $\lambda > 0$. When asset quality becomes more persistent, i.e., as $\lambda_L - \lambda_H$ increases, debt threshold d decreases.*[24]

The gap between high- and low-quality asset prices, $\Delta\phi$, creates an additional source of adverse selection. Since prices are forward looking, this second source of adverse selection is dynamic. Recall that in the i.i.d. case, increasing the debt threshold, tightens the high type's participation constraint. This new dynamic effect may create an opposing force and relax the high type's participation constraint. To see why this may happen, suppose high- and low-quality assets sell at different prices, i.e., $\Delta\phi = \phi_H - \phi_L > 0$, and the designer initially sets the debt face value at $D = \underline{s} + \phi_L$, which is completely safe regardless of asset quality. At $D = \underline{s} + \phi_L$, the high type strictly prefers to sell the debt tranche and the participation constraint does not bind. Increasing the debt threshold initially increases both ϕ_L and ϕ_H, and the low-quality asset price ϕ_L may increase faster than the high-quality asset price ϕ_H. As a result, the price gap $\Delta\phi$ may go down which lowers the adverse selection in asset prices and relaxes the high type's participation constraint. Despite this potentially countervailing force, Theorem 2 shows that condition (27) guarantees a unique security design equilibrium.

The following proposition shows that as persistence increases, the price difference $\Delta\phi$ - a proxy for price volatility in our model - increases and the dynamic effect becomes stronger.

Proposition 6. *Fix $\lambda > 0$. When asset quality becomes more persistent, i.e., as $\lambda_L - \lambda_H$ increases, $\Delta\phi$ increases.*

We observe that when persistence increases, consistent with the above proposition, condition (27) becomes more stringent.[25]

9 Implementation as a Repo Contract

Optimal securities derived in this paper describe contract terms on cashflows between borrowers and lenders upon realization of the state. In practice, the optimal security can be implemented in several ways. In this section, we demonstrate one prominent implementation that is a one-period repo contract traded in a pooling equilibrium, and a residual equity-like contract traded in a separating equilibrium. Furthermore, asymmetric information is an important friction for the bilateral repo market, especially in Europe. For example, Julliard et al. (2022) have shown that only 60% of bilateral repo contracts in

[24] If $d \in (\underline{s}, \bar{s})$ then as $\lambda_L - \lambda_H$ increases, d decreases strictly.

[25] Similarly, when agents become more patient, i.e., as β increases, price gap increases and condition (27) becomes more stringent as well.

the UK are backed by high quality collaterals. In this implementation, there is a representative borrower who values investors' inputs (at $z > 1$) more than investors, and hence has an incentive to purchase back the asset in every period to be able to use it to back securities in the next period. In this section, we first map the terms of repo contracts in the context of our model. These contract terms are endogenously determined given the underlying information and preference parameters. Next, we provide analytical solutions using a two-point distribution to link the primitives of the model to the repo contract terms.

9.1 Terms of the Repo Contract

When two parties enter a repo contract, one party sells an asset to another party at one price (which in our model corresponds to the loan value or the price of the debt tranche q_D) and commits to repurchase the same or another part of the same asset from the second party at a different price at a future date (which in our model corresponds to the face value of the debt tranche D). If the seller defaults during the life of the repo, the buyer (as the new owner) can sell the asset to a third party to offset the loss.[26] The most straightforward mapping of the optimal contract in the model to reality is as follows. During the term of the repo the lender receives s, which is the cash flow or the convenience yield/service flow from the asset (in this sense, lender is the legal owner) in an escrow account.[27] When the repo term is finished, there are two possibilities: (i) if the face value D is more than $s + \phi$, the borrower obtains the asset back from escrow by paying its price ϕ; (ii) if the face value D is less than $s + \phi$, then the borrower pays the lender remaining $D - s$, so that the lender obtains the promised face value D and the borrower takes the asset back from escrow.[28]

Our model complements the existing repo literature by offering an alternative explanation for why in a repo contract, an asset is sold and agreed to be repurchased.[29] This feature naturally arises in our

[26] We take the definition of a fixed term repo contract from the International Capital Market Association (ICMA).

[27] Escrow guarantees that the lender returns the asset. This is consistent with our model which focuses on limited commitment on the borrower side.

[28] According to this implementation, the pass through security follows the most common form of repo: the borrower sells the security to obtain inputs, the lender owns it via a custodian and consumes the benefit of being an owner which is s, the cash flow/service flow of the asset, and the borrower repurchases the security back at the end of the repo term (at price ϕ).

[29] The feature of asset repurchase is modeled differently in the repo literature. In Gottardi, Maurin, and Monnet (2017), asset repurchase arises from the need of lender and borrower to share risk since the collateral asset price is volatile. In Duffie (1996) and Parlatore (2019), the reason for asset repurchases comes from the illiquidity in the secondary market – if the secondary market is illiquid, it will be difficult for the borrowers to find the collateral asset to buy and hence they would like to repurchase the collateral asset back directly from the lender. In Bigio and Shi (2020), the asset repurchase option is introduced to meet the high-quality borrowers' incentive compatibility constraint.

model since borrowers always buy the collateral back at the end of borrowing -- hence, the repurchase leg endogenously arises in equilibrium.

We now describe the two terms of the repo contract: repo rate, r, and haircut, h. The definition of repo rate is straightforward:

$$r \equiv \frac{\text{face value}}{\text{loan value}} - 1 = \frac{D - q_D}{q_D}. \tag{29}$$

From the definition of repo rate r, we observe that the relationship between asset quality and interest rate is not straightforward because asset quality has two opposing effects on the repo rate. When asset quality worsens (improves), loan value is lower (higher), leading to a high (low) repo rate. In addition, the face value of the debt might be adjusted down (up), resulting in a lower (higher) repo rate.

To define the haircut, we first need to define the collateral value from the lender's perspective. In the context of our model, the lender expects to generate on average $E\phi/\beta$ from the sale of the collateral in the case of default, where $E\phi \equiv \lambda\phi_L + (1-\lambda)\phi_H$. We refer to this amount as the collateral value.[30]

The definition of repo haircut in our model is

$$h \equiv 1 - \frac{\text{loan value}}{\text{collateral value}} = 1 - \frac{q_D}{E\phi/\beta}. \tag{30}$$

From (16) we write the collateral value as:

$$E\phi/\beta = zq_D + \lambda z q_E + (1-\lambda)e_H, \tag{31}$$

where $e_H = \int_{d-\Delta\phi}^{\bar{s}} \widetilde{F}_H(s)ds$ is agent O's expected value of a high-quality equity tranche. Substituting (31) into (30), we obtain the following expression for haircut:

$$h = \underbrace{(z-1)\frac{q_D + \lambda q_E}{E\phi/\beta}}_{\text{gain from trade/collateral value}} + \underbrace{\frac{\lambda q_E + (1-\lambda)e_H}{E\phi/\beta}}_{\text{equity/collateral value}}. \tag{32}$$

The result from (32) shows that the repo haircut has two components. The first component arises because borrowers, who price the collateral asset, value the liquidity service of the asset to realize gains from trade, while lenders, who price the loan, do not value it. The term $z-1$ is the net marginal value of the liquidity service; it reflects heterogenous valuation over the collateral asset between lenders and borrowers in our model.[31] The second component is the value of the equity tranche relative to the collateral value and arises mechanically because the equity tranche is excluded from the repo debt.

[30] $E\phi$ is the end-of-period expected value of the collateral asset. Because the repo contract is an intra-period short-term contract, the collateral value in the definition of haircut refers to the beginning-of-period value, which equals $E\phi/\beta$.

[31] In the case of debt tranche as a passthrough security, the equity tranche disappears, and the haircut is $(z-1)/z$, solely driven by the marginal value of liquidity service.

9.2 Repo Contract with a Two-point Distribution

In this section, we illustrate the properties of the optimal repo contract in closed form when quality follows a two-point payoff distribution. We first consider the i.i.d. case and return briefly to the persistent case at the end of the section. The purpose of this exercise is to provide simpler expressions for the haircut and the repo rate with respect to the primitives of the model. Using these expressions we are able to find clean comparative statics of repo contract terms that generate empirically testable hypothesis, especially in the i.i.d. case which captures the characteristics of the vast treasury repo market in the US.

Suppose that the high (low) quality asset pays one unit of payoff with probability π_H (π_L) and pays zero otherwise where $0 \leq \pi_L < \pi_H \leq 1$ and $\lambda = \lambda_L = \lambda_H$. The debt contract takes a simple form. Regardless of the realization of the payoff, it pays the resale price ϕ. In addition, it pays d units if the current payoff is one.[32]

Let the expected value of the payoff (based on public information) be given by $Es \equiv (1-\lambda)\pi_H + \lambda\pi_L$. Expressions for repo rate and haircut given in (29) and (32) become much simpler. The repo rate is:

$$r = \frac{1 - Es}{\nu}, \tag{33}$$

and haircut is

$$h = 1 - \frac{\beta}{1 - \frac{Es}{\nu}}, \tag{34}$$

where $\nu \equiv \lambda(\pi_H - \pi_L)/(z-1)$ captures the degree of adverse selection. Severity of adverse selection increases in the probability that the asset is low quality (λ) and the difference in the probability of obtaining a positive payoff under high versus low quality ($\pi_H - \pi_L$) and decreases in gains from trade $(z-1)$. We observe from equation (34) that haircut is increasing in adverse selection ν holding Es fixed, and the sensitivity of haircut to adverse selection is increasing in β. The latter observation is another manifestation of the dynamic feedback between collateral price and contract terms: when agents become more forward looking, the role of resale price in backing the loan becomes more important. Hence, higher adverse selection lowers price and leads to a higher haircut. Comparative statics on haircut with

[32] The expressions for the terms of this repo contract are as follows:

$$\begin{aligned} d &= D - \phi = \frac{\frac{\beta}{1-\beta z}\left[z\lambda\pi_L + (1-\lambda)\pi_H\right]}{\frac{z}{z-1}\lambda\left(\pi_H - \pi_L\right) - \frac{1-\beta[1+\lambda(z-1)]}{1-\beta z}\pi_H} < 1, \\ \phi &= \frac{\beta}{1-\beta z}\left[z\lambda\pi_L + (1-\lambda)\pi_H + (1-\lambda)(z-1)\pi_H d\right]. \end{aligned}$$

respect to information friction that ignores the dynamic feedback and take resale price as exogenously given would then be inaccurate. The following proposition follows immediately from (33) and (34) and describes the comparative statics of repo rate and haircut and.

Proposition 7. *Both the repo rate and the haircut are decreasing in the expected value of the payoff based on public information, Es, holding ν fixed. The repo rate is decreasing and the repo haircut is increasing in the degree of adverse selection, ν, holding Es fixed.*

This proposition maps out how the degree of adverse selection and the expected value of the payoff, which are functions of the primitives of the model, affect repo rate and haircut. The only part of Proposition 7 that may seem counterintuitive is the statement that repo rate is decreasing in adverse selection. In fact, this result is in the same spirit as the standard result in credit rationing models (Stiglitz and Weiss (1981)). When adverse selection increases, haircut goes up, which means that the face value of repo loan is lower, making the repo loan safer and leading to a lower repo rate.

The results in this proposition indicate that the impact of adverse selection on repo terms is intricate. When testing how adverse selection affects repo rates and haircuts, empiricists need to control for changes in the expected value of the asset's payoff. The degree of adverse selection and the expected value of the payoff can be inferred from secondary information (such as prices, dividends, credit ratings, convenience yields, etc.). With these implementable metrics, the simple analytical solution provides new testable implications for cross-sectional repo contracts.

In segments of the repo market that use low-quality collaterals, private information advantage can be long-lived. Motivated by this observation, we next demonstrate the effect of persistent private information for the two-point distribution case. The main message is that higher persistence in private information leads to higher price volatility, lower collateral values and funding raised, larger haircuts and repo rates. To allow for persistence, we let $\lambda_L > \lambda_H$ where $\Delta\lambda = \lambda_L - \lambda_H$. To simplify the analysis and obtain closed-form solutions for the repo terms, we further assume that $\pi_H = 1$ and $\pi_L = 0$, i.e., the high quality asset always pays one unit of payoff, and the low-quality asset always pays zero. The following proposition describes the comparative statics for outcomes of economic interest as persistence in quality $\Delta\lambda$ increases holding the steady state quality distribution λ constant.

Proposition 8. *Keeping λ constant, as $\Delta\lambda$ increases (i) the debt threshold, d decreases; (ii) price volatility, $\Delta\phi$, increases; (iii) collateral value $E\phi$ decreases; (iv) loan value, q_D, decreases; (v) haircut, h, increases; (vi) repo rate, r, increases.*

We present the proof of Proposition 8 in the Online Appendix.

10 Conclusion

Our paper studies optimal security design in a dynamic lemons market. We demonstrate that one implementation of our optimal security design involves short-term collateralized debt. Because optimal security design helps coordinate investors' intertemporal decisions, the dynamic lemons market under optimal security design is robust to multiple equilibria induced by intertemporal miscoordination. We also explore the economic implications of an implementation of optimal security, short-term repos, and derive dynamic equilibrium properties of repo rates and haircuts. Our setup can be applied to any collateralized borrowing where the collaterals are traded in the capital market.[33] The underlying economic mechanism of our theory is the price liquidity feedback effect derived from the fact that collateral assets can be resold and resale prices can back security payments. Optimal security design eliminates multiplicity, generates greater amounts of liquidity, and restores the economy to a unique Pareto-optimal equilibrium.

According to the current understanding, the shadow banking system of overnight repurchase agreements, asset-backed securities, broker-dealers, and investments contributed to the Great Depression and the runs on the shadow banking system were classic bank runs à la Diamond and Dybvig (1983). However, this popular explanation ignores the fact that most securitized products and short-term funding instruments of these shadow banks are backed by the resale prices of the assets on their balance sheet (in addition to dividend/interest payments). Our model implies that in a dynamic economy, when financial intermediaries can flexibly tranche their assets, self-fulfilling price dynamics can be removed and the amount of funding liquidity as well as the real output in the economy will be greatly improved. Securitization in fact eliminates multiple equilibria and excessive volatility in asset prices and liquidity. Nevertheless, our theory identifies a new source of financial fragility that potentially emerges via the price-liquidity feedback. The expectation of low asset prices in the future could induce adverse selection in the present thus lead to a self-fulfilling low asset price equilibrium. Moreover, we find in a repo implementation of our model, that more persistence in private information results in more adverse selection, volatile asset prices, a lower amount of repo debt financing, exacerbating the credit crunch. We conclude, therefore, by pointing out that as the current global financial system moves from bank-based toward

[33] It has been observed that more firms raise funding and manage their working capital directly from investors by issuing securities backed by marketable collateral assets on their balance sheets, sidestepping banks or other traditional financial intermediaries. For instance, Apple Inc. reported $5.2 billion of repo borrowing in its 2020 10-K filing to support its working capital need during the COVID-19 pandemic. An implication of this practice is that firms now have incentives to acquire marketable assets (such as high-grade sovereign and corporate bonds) to access funding liquidity directly.

market-governed, understanding the dynamic feedback mechanism between asset prices and funding liquidity identified in this paper is critical. The price feedback mechanism in the market-based financial system may generate more funding and promote greater economic growth, but at the same time it is possible to ignite destabilizing self-fulfilling crises. Economic policy-makers and financial regulators need to closely monitor this new source of financial instability.

11 Data Availability Statement

There are no new data associated with this article. No new data were generated or analysed in support of this research.

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A Appendix

A.1 Proof of Proposition 1

Let $\overline{q} \equiv \lambda E_L y + (1-\lambda) E_H y$. Note that $z\overline{q} - E_H y \gtreqless 0$ iff $E_L y / E_H y \gtreqless \zeta$.

Consider the case $E_L y / E_H y > \zeta$. Suppose that the equilibrium price q is strictly less than $\overline{q}$. In this case an investor can deviate and bid $\overline{q} - \epsilon$ where $\epsilon > 0$. For ϵ small enough, $z\left(\overline{q} - \epsilon\right) - E_H y > 0$. Hence at this price both types sell the security and the deviation generates strictly positive surplus. This means that the equilibrium price must be at least $\overline{q}$. At price $\overline{q}$ or above both types sell the security, hence the only price that is consistent with zero profit condition is $q = \overline{q}$.

Now consider the case $E_L y / E_H y < \zeta$. In this case high type will sell the security only if q is sufficiently larger than $\overline{q}$. However, at prices above $\overline{q}$, investors make negative profit. Hence equilibrium price must be below $\overline{q}$. If q is below $\left(E_L y\right)/z$ then neither type sells the security. In this case, one of the investors can deviate and bid $E_L y - \epsilon$ where $\epsilon > 0$. For ϵ small enough, $z\left(E_L y - \epsilon\right) - E_L y > 0$ so the low type sells the security and the deviating agent makes strictly positive surplus. If q is at least $\left(E_L y\right)/z$ but less than $E_L y$ then the low type sells the security to the investors who bid that price. In this case, one of the investors who bids $E_L y$ or less can deviate and bid slightly above q. This agent then buys the security alone and increases her surplus. At prices greater than equal to $E_L y$ (and below $\overline{q}$), the low type alone sells the security. Hence the only price that is consistent with zero profit condition is $q = E_L y$.

A.2 Proof of Proposition 3

We first present a lemma that simplifies the analysis.

Lemma 1. *If two securities, y and y', are both traded in a pooling (separating) equilibrium, then $y + y'$ is also traded in a pooling (separating) equilibrium. Moreover, if a feasible security design contains y and y', replacing the two securities by $y + y'$ is also a feasible security design and the value of the designer's objective remains the same in these two cases. Hence, w.l.o.g. we can restrict attention to security designs that contain at most two securities, one traded in a pooling equilibrium and the other traded in a separating equilibrium.*

Proof. If two securities, y and y', are both traded in a pooling equilibrium, $E_L y \geq \zeta E_H y$ and $E_L y' \geq \zeta E_H y'$. Then combining these two securities results in a security traded in a pooling equilibrium. Similarly, combining two securities traded in a separating equilibrium results in a security traded in a separating equilibrium. To see the second statement in the lemma, first note that replacing the two securities with their combination is clearly feasible. In addition, when y, y' and $y + y'$ all trade in a pooling (separating) equilibrium, q'', the price of $y + y'$, is the sum of q and q', the prices of y and y'. Now consider the pooling case. Ignoring the irrelevant terms, agent O's payoff when the two securities are separate is:

$$\lambda \int \{a [zq - y(s)] + a [zq' - y'(s)]\} \, dF_L(s) + (1 - \lambda) \int \{a [zq - y(s)] + a [zq' - y'(s)]\} \, dF_H(s)$$

and when they are combined is:

$$\lambda \int \{a [zq'' - (y(s) + y'(s))]\} \, dF_L(s) + (1 - \lambda) \int \{a [zq'' - (y(s) + y'(s))]\} \, dF_H(s).$$

Since $q'' = q + q'$, when the securities are combined agent O's payoff is unchanged.

Next consider the separating case. Once again ignoring the irrelevant terms, agent O's payoff when the two securities are separate is:

$$\lambda \int \{a [zq - y(s)] + a [zq' - y'(s)]\} \, dF_L(s) + (1 - \lambda) \int \{ay(s) + ay'(s)\} \, dF_H(s)$$

and when they are combined is:

$$\lambda \int \{a [zq'' - (y(s) + y'(s))]\} \, dF_L(s) + (1 - \lambda) \int \{a (y(s) + y'(s))\} \, dF_H(s).$$

Once again, when the securities are combined agent O's payoff is unchanged. $\square$

Using Lemma 1 we restate the optimal security design problem as choosing the pooling tranche of the asset, $y_D(s)$, to maximize the value of a high-quality debt tranche:

$$E_H y_D(s) \tag{A.1}$$

subject to

$$s + \phi - y_D(s) \geq 0, \forall s \in [\underline{s}, \bar{s}], \tag{A.2}$$

$$E_L y_D(s) - \zeta E_H y_D(s) \geq 0, \tag{A.3}$$

and

$$y_D(s) \geq y_D(s') \text{ if } s \geq s', \forall s \in [\underline{s}, \bar{s}]. \tag{A.4}$$

We obtain the objective function by plugging the security prices and the quantities given in Proposition 1 into the designer's objective. The first constraint above is the simplified feasibility constraint (2) and requires $y_D(s)$ to be backed by the underlying asset. The second is the requirement in Proposition 1 that the security is sold in a pooling equilibrium. The third constraint restates (1) that requires the pooling security to be monotone.[34][35]

Observe that any right-continuous monotone security $y(s)$ taking values in $[\underline{s} + \phi, \bar{s} + \phi]$ as:

$$y(s) = \phi + \underline{s} + \int_{\underline{s}}^{\bar{s}} \chi(j)dj,$$

where $\chi(j) \geq 0$ for all $j \in [\underline{s}, \bar{s}]$. Then,

$$E_Q y = \phi + \underline{s} + \int_{\underline{s}}^{\bar{s}} \widetilde{F}_Q(j)\chi(j)dj.$$

The optimization problem (A.1) is equivalent to the following problem:

$$\arg\max_{\chi \geq 0} \int_{\underline{s}}^{\bar{s}} \widetilde{F}_H(x)\chi(x)dx, \tag{A.5}$$

$$s.t. \underline{s} + \int_{\underline{s}}^{x} \chi(j)dj \leq x, \forall x \in [\underline{s}, \bar{s}], \tag{A.6}$$

$$\int_{\underline{s}}^{\bar{s}} \left[\widetilde{F}_L(x) - \zeta\widetilde{F}_H(x)\right]\chi(x)dx + (1-\zeta)(\underline{s} + \phi) \geq 0, \tag{A.7}$$

$$\chi(x) \geq 0, \forall x \in [\underline{s}, \bar{s}] \tag{A.8}$$

[34]Equation (1) needs to hold for the residual equity tranche as well but this constraint is not binding.

[35]The uniqueness of equilibrium does not depend on the restriction of issuing monotone securities, and also holds when the borrower issues Arrow securities against the dividend payment and the resale value of the asset. This result is available upon request.

Note that the feasible set is compact, convex and nonempty so the optimization problem must have a solution. Moreover, since the objective function is bounded above, the solution must be finite. The Lagrangian of the optimization problem is

$$\begin{aligned}
\mathcal{L}\left(x;\gamma,\mu,\mu_{\chi}\right) &= \int_{\underline{s}}^{\bar{s}} \widetilde{F}_H(x)\chi(x)dx + \int_{\underline{s}}^{\bar{s}} \gamma(x)\left[x-\underline{s}-\int_{\underline{s}}^{x}\chi(j)dj\right]dx \\
&+\mu\left\{\int_{\underline{s}}^{\bar{s}}\left[\widetilde{F}_L(x)-\zeta\widetilde{F}_H(x)\right]\chi(x)dx+(1-\zeta)\left(\underline{s}+\phi\right)\right\}+\int_{\underline{s}}^{\bar{s}}\mu_{\chi}(x)\chi(x)dx \\
&= \int_{\underline{s}}^{\bar{s}}\left\{\widetilde{F}_H(x)+\mu\left[\widetilde{F}_L(x)-\zeta\widetilde{F}_H(x)\right]-\eta\left(x\right)+\mu_{\chi}(x)\right\}\chi(x)dx \\
&+\mu(1-\zeta)\left(\underline{s}+\phi\right)+\int_{\underline{s}}^{\bar{s}}\eta\left(x\right)dx,
\end{aligned}$$

where the second equality is obtained by using integration by parts on the second term of the Lagrangian, and then setting $\eta\left(x\right)=\int_{x}^{\bar{s}+\phi}\gamma(j)dj$. Let $\mathcal{L}^{*}=\min_{\gamma\geq 0,\mu\geq 0,\mu_{\chi}\geq 0}\left[\max_{\chi\geq 0}\mathcal{L}\left(x;\gamma,\mu,\mu_{\chi}\right)\right].$ Note that $\mathcal{L}^{*}$ is the value of the original optimization problem. The quantity inside the curly brackets must be zero or otherwise the value of the optimization problem would be infinite. Consider the following problem,

$$\begin{aligned}
\min_{\mu\geq 0}\quad \min\nolimits_{\eta\geq 0,\mu_{x}\geq 0}\quad & \mu(1-\zeta)\left(\underline{s}+\phi\right)+\int_{\underline{s}}^{\bar{s}}\eta\left(x\right)dx \\
s.t.\qquad & \widetilde{F}_H(x)+\mu\left[\widetilde{F}_L(x)-\zeta\widetilde{F}_H(x)\right]-\eta\left(x\right)+\mu_{\chi}(x)=0.
\end{aligned}$$

The value of this problem is $\mathcal{L}^{*}$. Let $H_{\mu}\left(x\right)=\widetilde{F}_H(x)+\mu\left[\widetilde{F}_L(x)-\zeta\widetilde{F}_H(x)\right]$, and rewrite one more time as:

$$\begin{aligned}
\min_{\mu\geq 0}\quad \min\nolimits_{\eta\geq 0}\quad & \mu(1-\zeta)\left(\underline{s}+\phi\right)+\int_{\underline{s}}^{\bar{s}}\eta\left(x\right)dx \\
s.t.\qquad & \eta\left(x\right)\geq H_{\mu}\left(x\right),
\end{aligned}$$

and the constraint that $\eta(x)$ is a decreasing function in x. Note, $h_{\mu}\left(x\right)\equiv\frac{\partial H_{\mu}(x)}{\partial x}=-\left[f_H\left(x\right)+\mu\left(f_L\left(x\right)-\zeta f_H\left(x\right)\right)\right]$, $H_{\mu}\left(\underline{s}\right)=1+\mu\left[1-\zeta\right]>0$ and $H_{\mu}\left(\bar{s}\right)=0$.

Proof. If $\mu=0$ then $h_{\mu}\left(x\right)<0$. If $\mu>0$ then

$$h_{\mu}\left(x\right)=-f_H\left(x\right)\left[1+\mu\left(\frac{f_L\left(x\right)}{f_H\left(x\right)}-\zeta\right)\right].$$

Since $f_L\left(x\right)/f_H\left(x\right)$ is decreasing for $x\in\left[\underline{s},\bar{s}\right]$, $h_{\mu}\left(x\right)$ can change sign from negative to positive only once over $\left[\underline{s},\bar{s}\right]$. Since h_{μ} changes its sign at most once from negative to positive, and since $H_{\mu}\left(\underline{s}\right)>0$ and $H_{\mu}\left(\bar{s}\right)=0$, either there exists a unique $x_{\mu}^{*}\in\left(\underline{s},\bar{s}\right)$ such that $H_{\mu}\left(x_{\mu}^{*}\right)=0$, or $H_{\mu}\left(x\right)>0$ for all $x\in\left(\underline{s},\bar{s}\right)$. In the latter case, we let $x_{\mu}^{*}=\bar{s}$.

Note that for given $\mu \geq 0$ optimal η_μ is given by:

$$\eta_\mu(x) = \begin{cases} H_\mu(x) & \text{if} \quad x \leq x_\mu^* \\ 0 & \text{if} \quad x > x_\mu^* \end{cases}.$$

Plugging this into the minimization problem we get:

$$\min_{\mu \geq 0} \mu(1-\zeta)\phi + \int_{\underline{s}}^{x_\mu^*} \left(\widetilde{F}_H(x) + \mu \left[\widetilde{F}_L(x) - \zeta \widetilde{F}_H(x) \right] \right) dx.$$

The first order condition for this problem is:

$$(1-\zeta)\phi + \int_{\underline{s}}^{x_\mu^*} \left[\widetilde{F}_L(x) - \zeta \widetilde{F}_H(x) \right] dx + \frac{\partial x_\mu^*}{\partial \mu} H_\mu \left(x_\mu^* \right) \geq 0$$

Because $H_\mu \left(x_\mu^* \right) = 0$,

$$(1-\zeta)\phi + \int_{\underline{s}}^{x_\mu^*} \left[\widetilde{F}_L(x) - \zeta \widetilde{F}_H(x) \right] dx \geq 0$$

with complementary slackness.

Let $x^* \in (\underline{s}, \bar{s}]$ be the unique s for which

$$(1-\zeta)\phi + \int_{\underline{s}}^{x^*} \left[\widetilde{F}_L(x) - \zeta \widetilde{F}_H(x) \right] dx = 0$$

if it exists. If

$$(1-\zeta)\phi + \int_{\underline{s}}^{\bar{s}} \left[\widetilde{F}_L(x) - \zeta \widetilde{F}_H(x) \right] dx > 0$$

for all $x \in [\underline{s}, \bar{s}]$, then $x^* = \bar{s}$.

If $x^* < \bar{s}$ then $\mu > 0$, $x_\mu^* = x^*$, and

$$\mathcal{L}^* = \mu(1-\zeta)\phi + \int_{\underline{s}}^{x^*} \left(\widetilde{F}_H(x) + \mu \left[\widetilde{F}_L(x) - \zeta \widetilde{F}_H(x) \right] \right) dx = \int_{\underline{s}}^{x^*} \widetilde{F}_H(s) ds.$$

If $x^* = \bar{s}$ then $\mu = 0$, $x_\mu^* = \bar{s}$, and

$$\mathcal{L}^* = \int_{\underline{s}}^{\bar{s}} \widetilde{F}_H(s) ds.$$

To complete the proof, let $D = x^* + \phi$ and note that $\chi(x) = 1$ for $x \in [\underline{s}, D-\phi)$ and $\chi(x) = 0$ for $x \in [D-\phi, \bar{s}]$ achieves the value $\mathcal{L}^*$ and it is feasible, and must be optimal for the original problem. $\square$

A.3 Proof of Theorem 1

Using Proposition 3, we write the designer's problem as:

$$\max_{d \in [\underline{s}, \bar{s}]} \int_{\underline{s}}^{d} \widetilde{F}_H(s) ds \tag{A.9}$$

subject to

$$\underline{s}+\phi+\int_{\underline{s}}^{d}\widetilde{F}_L(s)ds-\zeta\left(\underline{s}+\phi+\int_{\underline{s}}^{d}\widetilde{F}_H(s)ds\right)\geq 0. \tag{A.10}$$

To obtain (A.9) and (A.10), we substitute (13) into the designer's objective given in (A.1) and into (C.11) which guarantees that the debt tranche is sold in a pooling equilibrium. Observe that to maximize (C.21) designer must set d as large as possible subject to satisfying the constraint (A.10). We first show that either there is a unique d that satisfies (A.10) with equality, or (A.10) is not binding. Let

$$\mathcal{T}(x)\equiv(z-1)\left(\underline{s}+\phi+\int_{\underline{s}}^{d}\widetilde{F}_H(s)ds\right)-\lambda z\left(\int_{\underline{s}}^{d}\left[\widetilde{F}_H(s)-\widetilde{F}_L(s)\right]ds\right)$$

Observe that,

$$\mathcal{T}(\underline{s})=(z-1)\left(\phi+\underline{s}\right)>0, \qquad \mathcal{T}'(x)=(z-1)\widetilde{F}_H(x)-z\lambda\left[\widetilde{F}_H(x)-\widetilde{F}_L(x)\right],$$

$$\mathcal{T}'(\underline{s})=z-1>0, \qquad \mathcal{T}'(\overline{s})=0,$$

$$\mathcal{T}''(x)=-(z-1)f_H(x)+z\lambda\left[f_H(x)-f_L(x)\right]=f_H(x)\left[z(\lambda-1)+1-z\lambda\frac{f_L(x)}{f_H(x)}\right].$$

When $\frac{f_L(x)}{f_H(x)}$ is monotonically decreasing in s, $\mathcal{T}(x)$ is quasi-concave with $\mathcal{T}(\underline{s})>0$. So, there is either a unique d that satisfies $\mathcal{T}(d)=0$ or $\mathcal{T}(x)>0$ for all $x\in[\underline{s},\overline{s}]$.

Case (i): Constraint (A.10) is binding. In this case the face value of the debt contract that solves the security design problem is the unique solution to $\mathcal{T}(d)=0$:

$$\phi=\frac{z}{z-1}\lambda\int_{\underline{s}}^{d}\left[\widetilde{F}_H(s)-\widetilde{F}_L(s)\right]ds-\int_{\underline{s}}^{d}\widetilde{F}_H(s)ds-\underline{s}. \tag{A.11}$$

In addition, the asset price ϕ satisfies (16). Substituting for q_D and q_E we rewrite (16) as:

$$\phi=\frac{\beta}{1-\beta z}\left\{z\left[\lambda E_L s+(1-\lambda)E_H s\right]-(1-\lambda)(z-1)\int_{\underline{s}}^{d}\widetilde{F}_H(s)ds\right\}. \tag{A.12}$$

Substituting ϕ in (A.11) using (A.12), the equilibrium can be solved by a single equation of d, $\Gamma(d)=0$, where

$$\begin{aligned}\Gamma(d)=&\frac{\beta}{1-\beta z}\left\{z\left[\lambda E_L s+(1-\lambda)E_H s\right]-(1-\lambda)(z-1)\int_{\underline{s}}^{d}\widetilde{F}_H(s)ds\right\}\\&-\frac{z}{z-1}\lambda\int_{\underline{s}}^{d}\left[\widetilde{F}_H(s)-\widetilde{F}_L(s)\right]ds+\int_{\underline{s}}^{d}\widetilde{F}_H(s)ds+\underline{s}\end{aligned}$$

Observe that:

$$\begin{aligned}
\Gamma'(d) &= \frac{\beta}{1-\beta z}(1-\lambda)(z-1)\widetilde{F}_H(d) - \frac{z}{z-1}\lambda\left[\widetilde{F}_H(d)-\widetilde{F}_L(d)\right] + \widetilde{F}_H(d) \\
&= \left[\frac{\beta}{1-\beta z}(1-\lambda)(z-1) + 1 - \frac{z}{z-1}\lambda\right]\widetilde{F}_H(d) + \frac{z}{z-1}\lambda\widetilde{F}_L(d). \\
\Gamma''(d) &= -\left[\frac{\beta}{1-\beta z}(1-\lambda)(z-1) + 1 - \frac{z}{z-1}\lambda\right] f_H(d) - \frac{z}{z-1}\lambda f_L(d) \\
&= f_H(d)\left\{\frac{z}{z-1}\lambda\left[1-\frac{f_L(d)}{f_H(d)}\right] - \frac{\beta}{1-\beta z}(1-\lambda)(z-1) - 1\right\} \\
\Gamma(\underline{s}) &= \underline{s}\left[1+\frac{\beta}{1-\beta z}(1-\lambda)(z-1)\right] + \frac{\beta}{1-\beta z}\left[z\lambda E_L s + (1-\lambda)E_H s\right] > 0 \\
\Gamma'(\underline{s}) &= \frac{\beta}{1-\beta z}(1-\lambda)(z-1) + 1 > 0 \\
\Gamma'(\overline{s}) &= 0.
\end{aligned}$$

Once again $\Gamma(s)$ is quasi-concave if $\frac{f_L(d)}{f_H(d)}$ is monotonically decreasing in d. Because $\Gamma(\underline{s}) > 0$, there is a unique equilibrium. The constraint (C.22) is binding iff $\Gamma(\overline{s}) < 0$. We rewrite $\Gamma(\overline{s})$ as:

$$\begin{aligned}
\Gamma(\overline{s}) &= \frac{\beta z}{1-\beta z}\left[\lambda E_L s + (1-\lambda)E_H s\right] - \frac{z}{z-1}\lambda\int_{\underline{s}}^{\overline{s}}\left[\widetilde{F}_H(s)-\widetilde{F}_L(s)\right]ds + \int_{\underline{s}}^{\overline{s}}\widetilde{F}_H(s)ds + s_L \\
&= \frac{E_H s}{(1-\beta z)(z-1)}\left[\lambda z(1-\beta)\left(\frac{E_L s}{E_H s}-1\right) + z - 1\right].
\end{aligned}$$

Hence, $\Gamma(\overline{s}) < 0$ if and only if

$$\frac{E_L s}{E_H s} < 1 - \frac{z-1}{z\lambda(1-\beta)} = \kappa_P.$$

Case (ii): Constraint (A.10) is not binding iff $\frac{E_L s}{E_H s} \geq \kappa_P$. Then $d = \overline{s}$ and $\phi = \frac{\beta z}{1-\beta z}\left[\lambda E_L s + (1-\lambda)E_H s\right]$.