Can Competition Increase Profits in Factor Investing?*

Victor DeMiguel Alberto Martín-Utrera Raman Uppal

June 29, 2022

Abstract

The increasing number of institutions exploiting factor-investing strategies raises concerns that competition may erode profits. We use a game-theoretic model to show that, while competition among investors exploiting a particular factor erodes profits because of the negative externality of their price impact on each other, competition to exploit *other* factors can increase profits from the first factor because of the positive externality from *trading diversification* (netting of trades across factors). We calibrate our model using the investment and profitability factors and find that competition to exploit the profitability factor leads to a 68% increase in the capacity and a 143% increase in the profit from the investment factor.

Keywords: capacity of quantitative strategies, crowding, price impact.
JEL Classification: G11, G12, G23, L11.

*DeMiguel, London Business School, e-mail: avmiguel@london.edu; Martín-Utrera, Iowa State University, e-mail: amutrera@iastate.edu; Uppal, EDHEC Business School, EDHEC Scientific Beta Research Chair, and CEPR, e-mail: raman.uppal@edhec.edu. We are grateful for comments from Noel Amenc, Simcha Barkai, Jean-Pierre Benoît, Jonathan Berk, Jules van Binsbergen, Svetlana Bryzgalova, Laurent Calvet, João Cocco, Thomas Dangl, James Dow, David Feldman (discussant), Gavin Feng, Francesco Franzoni, Thierry Foucault, Felix Goltz, Francisco Gomes, Amit Goyal, Christopher Hennessy, Nicholas Hirschey, Christian Heyerdahl-Larsen, Bruce Jacobs, Apostolos Kourtis, Augustin Landier, Hugues Langlois (discussant), Ashley Lester, Ken Levy, Jiacui Li, Abraham Lioui, Dong Lou, Christian Lundblad (discussant), Raphael Markellos, Toby Moskowitz (discussant), Narayan Naik, Vasant Naik, Emre Ozdenoren, Anna Pavlova, Jeffrey Pontiff, Scott Richardson, Vania Stavrakeva, Nikos Tessaromatis, David Thesmar, Cristian Tiu (discussant), Afonso Varatojo, Josef Zechner, Alan Zhang (discussant), and seminar participants at the 12th Annual Hedge Fund Research Conference, 27th Finance Forum, 2020 INFORMS Annual Meeting, 2020 NFA Conference, 4th SAFE Market Microstructure Conference, 2021 AFA Annual Meeting, 2021 FMA Annual Meeting, EDHEC Business School, INSEAD, King's Business School, Iowa State University, London Business School, Norwich Business School, SKEMA Business School, Schroeders, SYZ Asset Management, Université Catholique de Louvain, and University of St. Gallen. An earlier version of this paper was circulated under the title "What Alleviates Crowding in Factor Investing?"

1 Introduction

Factor investing is a low-cost approach to active fund management that exploits stock characteristics such as value, investment, and profitability. Although assets under management in factor investing have grown rapidly at 30% per annum since 2010 (Ratcliffe, Miranda, and Ang, 2017), reaching \$1.9 trillion by 2018 (BlackRock, 2021), the capacity of these strategies remains limited by price-impact costs. Indeed, there is a literature that characterizes a strategy's capacity, defined as the total investment that can be allocated to it before price-impact costs erode its profits entirely (Novy-Marx and Velikov, 2016; Frazzini, Israel, and Moskowitz, 2018; Chen and Velikov, 2021). Importantly, the growth in factor investing has been accompanied by a substantial increase in the *number* of institutions competing to exploit these strategies. For instance, 145 managers launched factor-investing products in 2018 (Flood, 2019). This raises concerns about *crowding*: as an increasing number of institutions exploit the same characteristic, competition leads them to overinvest and price-impact costs erode profits.[1] Our key contribution is to use a game-theoretic model to show that, while competition among investors exploiting a particular characteristic erodes profits because of the negative externality of their price impact on each other, competition to exploit *other* characteristics increases the profits from the first characteristic because of the positive externality from *trading diversification* (netting of trades across characteristics).

To pave the way for our analysis of the effect of competition in the presence of trading diversification, our first contribution is to characterize theoretically and empirically the trading-diversification mechanism for the case with a single investor, that is, ignoring competition. Intuitively, a single investor can reduce price-impact costs by combining several characteristics whose portfolio-rebalancing trades are negatively correlated because their trades net out on average. However, we show theoretically that combining characteristics may reduce price-impact costs *even* when their rebalancing trades are not negatively correlated. Empirically, we consider a single investor exploiting 18 characteristics and find that there is

[1]For example, in his AFA presidential address Stein (2009) argues that "basic economic logic suggests that as more money is brought to bear against a given trading opportunity, any predictable excess returns must be reduced and eventually eliminated." Similarly, Jacobs and Levy (2014) state that: "Smart beta strategies are often based on common, generic factors used by many managers. This approach leaves their performance susceptible to factor crowding: Too many investors are buying (or selling) the same securities on the basis of the same factors."

a reduction of 16% in price-impact costs when trading them in combination rather than in isolation. Moreover, exploiting the 18 characteristics in combination leads to an increase in total capacity of 45%, from 239 to \$345 billion, in total optimal investment of 43%, from 116 to \$165 billion, and in total annual profits of 22%, from 1.5 to \$1.8 billion.

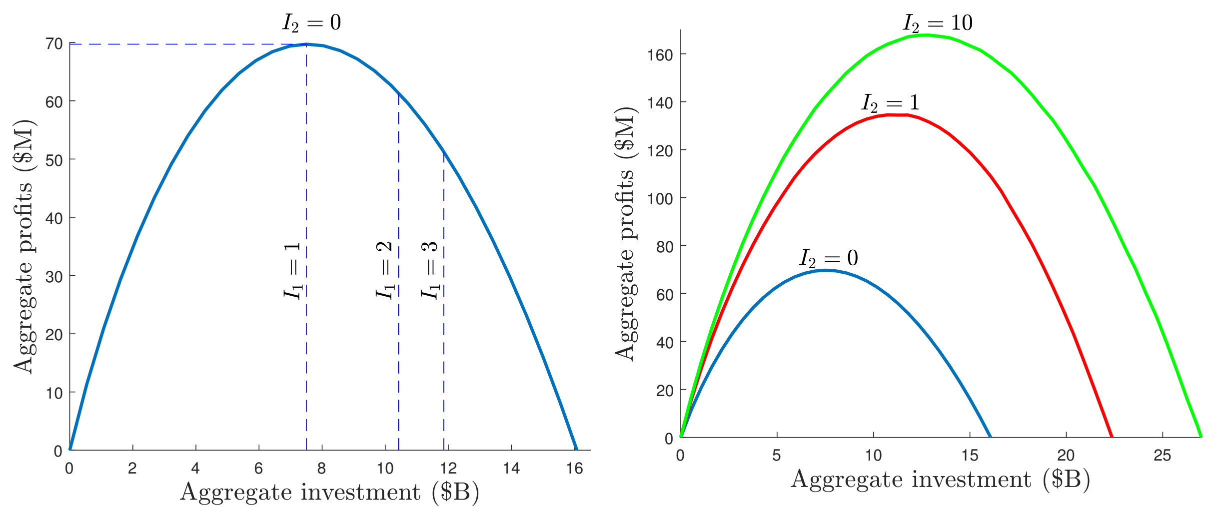
Our second contribution is to study the effect of competition in the presence of trading diversification. To do this, we develop a game-theoretic model related to the models in Berk and Green (2004) and Pástor and Stambaugh (2012), who consider competition among fund managers facing diseconomies of scale at the *fund* and *industry* levels, respectively. In contrast, we consider a model with two groups of factor investors, each group exploiting a different characteristic. Investors within each group compete to exploit the same characteristic and thus face diseconomies of scale at the *characteristic* level driven by price-impact costs. In particular, there is a negative externality within each group of investors because they do not internalize the effect that their trades exert on each other's price-impact cost. However, there is also a *positive externality* between the two groups of investors because they reduce each other's price-impact costs as a result of trading diversification across characteristics.

We provide closed-form expressions for the equilibrium investment positions and profits in the game and study how competition in the presence of trading diversification affects them. To gauge the magnitude of the effect, we also calibrate the model using "investment (asset growth)" as the first characteristic and "gross profitability" as the second. Figure 1 illustrates our main findings. Panel (a) illustrates the effect of competition to exploit a particular characteristic by depicting the aggregate profits of the investors exploiting the first characteristic as a function of their aggregate investment, when there are *no* investors exploiting the second characteristic. The graph shows that in this case the capacity of the first characteristic is around \$16 billion. Moreover, when there is a single investor exploiting the first characteristic ($I_1 = 1$), she maximizes her profits by investing around *half* of the characteristic's capacity. However, as the number of investors in the first characteristic I_1 increases, competition leads them to overinvest and, as a result, their aggregate profits decrease. In the limit, as the number of investors goes to infinity, their aggregate investment position converges to the strategy's capacity and their aggregate profits converge to zero because of price-impact costs. Thus, we obtain the intuitive result for the base case that

Figure 1: Competition and profits in factor investing

This figure illustrates the effect of competition on capacity, investment positions, and profits in the presence of trading diversification. The figure is calibrated using "investment (asset growth)" as the first characteristic and "gross profitability" as the second. For each panel, the horizontal axis depicts aggregate investment in the first characteristic (billions of dollars) and the vertical axis depicts aggregate annual profits from the first characteristic (millions of dollars). Panel (a) illustrates the effect of competition to exploit the same characteristic by depicting the aggregate profits of the investors exploiting the first characteristic as a function of their aggregate investment, when no investors are exploiting the second characteristic ($I_2 = 0$). The vertical dashed lines mark the optimal aggregate investment in the first characteristic when there are $I_1 = 1, 2, 3$ investors exploiting it. Panel (b) illustrates the effect of competition among investors exploiting the second characteristic by comparing the aggregate profits of investors exploiting the first characteristic for the cases where: (i) there are no investors exploiting the second characteristic ($I_2 = 0$, blue line) and thus there is no trading diversification, (ii) there is one investor ($I_2 = 1$, red line), and (iii) there are ten investors ($I_2 = 10$, green line) exploiting the second characteristic.

(a) Competition in first characteristic (b) Competition in second characteristic



competition among investors exploiting the *same* characteristic erodes their profits because of the negative externality of their price impact on each other.

Panel (b) of Figure 1 illustrates the effect of competition among investors exploiting the second characteristic in the presence of trading diversification. The graph compares the aggregate profits of investors exploiting the first characteristic for the cases where: (i) there are no investors exploiting the second characteristic ($I_2 = 0$, blue line) and thus there is no trading diversification, (ii) there is one investor ($I_2 = 1$, red line), and (iii) there are ten investors ($I_2 = 10$, green line) exploiting the second characteristic. Comparing the case of $I_2 = 0$ with that of $I_2 = 1$, we observe that the presence of an investor exploiting the second characteristic increases the capacity of the first characteristic from 16 to around \$22 billion and it also substantially increases the aggregate profits of investors exploiting the first characteristic. Thus, investment in the second characteristic increases profits from the first characteristic because of trading diversification.

Moreover, Panel (b) also shows that an increase in the number of investors exploiting the second characteristic from $I_2 = 1$ to $I_2 = 10$, further increases the capacity and aggregate profits from the first characteristic. *Thus, an increase in competition among investors exploiting the second characteristic further increases profits from the first characteristic.* Overall, comparing the case without investors exploiting the second characteristic ($I_2 = 0$) to the case with ten investors ($I_2 = 10$), competition among investors exploiting the second characteristic leads to a 68% increase in the capacity and a 143% increase in the maximum aggregate profit from the *first* characteristic.

Our paper has implications for the active debate about the extent to which price-impact costs constrain the capacity of quantitative strategies based on firm characteristics. While academic papers based on publicly available data suggest that trading costs constrain capacity substantially (Korajczyk and Sadka, 2004; Novy-Marx and Velikov, 2016; Chen and Velikov, 2021), research based on proprietary data from large money managers such as AQR (Frazzini, Israel, and Moskowitz, 2015) and BlackRock (Ratcliffe et al., 2017) suggests that the price-impact costs incurred by these institutions are much smaller, and thus the capacity of quantitative strategies is much larger than previously thought. Our work presents a channel that reconciles these contrasting views: financial institutions using quantitative strategies may not face the level of price-impact costs documented in the academic literature because they benefit from trading diversification—either by exploiting different characteristics in combination within their own operation, or by exploiting a particular characteristic while other institutions are taking advantage of different characteristics.

Our work has implications also for the industrial organization of the quantitative-investment industry. For example, financial institutions should focus not only on characteristics that are profitable, but also are exploited by a relatively *small number* of institutions. This intuitive implication of our work may help to explain the high degree of concentration in the investment-management industry, with just three firms—BlackRock, Vanguard, and State Street—holding around 80% of the assets in ETF products (Baert, 2018). Moreover, institutions have an incentive to acquire competitors—a strategy recently adopted by Invesco (Carlson, 2019)—in order to both reduce the intensity of competition and exploit the trading-diversification benefits from combining a broader range of strategies in house.

Our work is related to the mutual-fund literature on competition; Berk and van Binsbergen (2017) provide a comprehensive review of this literature. The seminal paper by Berk and Green (2004) considers managers who face diseconomies of scale at the *fund* level, while Pástor and Stambaugh (2012) assume diseconomies of scale at the *industry* level.[2] In contrast, we consider diseconomies of scale at the *characteristic* level and provide a microfoundation for them based on price-impact costs, which we estimate using the model of Frazzini et al. (2018). This microfoundation is consistent with Edelen, Evans, and Kadlec (2007) and Pástor, Stambaugh, and Taylor (2015), who show that trading costs are the primary source of diseconomies of scale.[3] We contribute to this literature by showing that competition among investors exploiting *different* characteristics can *alleviate* the diseconomies of scale in fund management because of trading diversification.

There is also a mutual-fund literature on the impact on characteristic returns of nondiscretionary trading (Lou, 2012). Li (2021) shows that nondiscretionary trading leads to substantial price impact on the size and value characteristics. In contrast, we focus on how *competition* among investors affects price-impact costs, and thus, instead of relying on measures of nondiscretionary trading, in Appendix C we use mutual-fund data to study the impact of a novel measure of buying and selling competition on characteristic returns.

Our work is related to Bonelli, Landier, Simon, and Thesmar (2019), who consider investors competing to exploit a *single* characteristic and analyze how capacity and performance depend on the *characteristic persistence* and the traders' estimates of the number of

[2]In addition, Wahal and Wang (2011) find that incumbent funds that have high overlap in holdings with entrant funds reduce management fees and earn lower alphas, Hoberg, Kumar, and Prabhala (2018) show that buy-side competition among mutual funds explains future alphas, and Feldman, Saxena, and Xu (2020, 2021) show that when industry concentration is lower, net alpha and industry size are smaller.

[3]Edelen et al. (2007) state that "We estimate annual trading costs for a large sample of equity funds and find that they are comparable in magnitude to the expense ratio; that they have higher cross-sectional variation that is related to fund trade size; and that they have an increasingly detrimental impact on performance as the fund's relative trade size increases. Moreover, relative trade size subsumes fund size in regressions of fund returns, which suggests that trading costs are the primary source of diseconomies of scale for funds." Pástor et al. (2015) explain that "evidence is mounting that trading by mutual funds is capable of exerting meaningful price pressure in equity markets" and cite six papers in their Footnote 2 that support this claim. In their own analysis, Pástor et al. (2015) find that "the negative relation between industry size and fund performance is stronger for funds with higher turnover and volatility as well as small-cap funds. These results seem sensible because funds that are aggressive in their trading, and funds that trade illiquid assets, see their high trading costs reap smaller profits when competing in a more crowded industry."

competitors. In contrast, we consider investors competing to exploit *multiple* characteristics and analyze how *trading diversification* affects capacity and performance.

Other researchers have also found that combining characteristics reduces transaction costs. Barroso (2012) and Barroso and Santa-Clara (2015) explain that transaction costs depend on the "interaction between characteristics." Novy-Marx and Velikov (2016) point out that investors trading one strategy can opportunistically take small positions in another at negative trading costs. DeMiguel, Martin-Utrera, Nogales, and Uppal (2020) demonstrate that "combining characteristics allows one to diversify trading." Detzel, Novy-Marx, and Velikov (2021) show that this trading-diversification effect leads to increases of up to 30% in the squared Sharpe ratio of portfolios implied by prominent asset-pricing models. Two features distinguish our work from this literature. First, most papers focus on the case with *proportional* transaction costs, and thus, they cannot study the impact of trading diversification on strategy capacity and optimal investment positions.[4] In contrast, we focus on the case with price-impact costs and demonstrate that trading diversification has a substantial effect on capacity and investment positions. Second, and more importantly, while the aforementioned papers consider the case with a single investor, we consider the case with multiple competing investors and show how the *strategic interactions* among financial institutions can alleviate crowding concerns in factor investing because of trading diversification.

Our work is also related to a growing literature on crowding. Harvey, Liu, Tan, and Zhu (2020) study the impact of team management on the crowding of ideas in discretionary funds. Chincarini (2017) finds that, when portfolio managers consider price impact, they hold less crowded portfolios because they prefer to "trade very small amounts of many more stocks." Lou and Polk (2022) show that crowded momentum trading leads to lower long-term momentum returns and Hoberg, Kumar, and Prabhala (2020) show that the momentum portfolio produces alpha only when it is constructed from stocks held by funds that do not

[4]A notable exception is Frazzini et al. (2015), who focus on the case with price-impact costs and show that the capacity of the equally weighted combination of value and momentum is larger than that of each of the two strategies in isolation. In contrast, we consider the *optimal* combination of 18 characteristics and find that their capacity in combination is 45% larger than their *aggregate* capacity in isolation. Also, DeMiguel et al. (2020) consider in section IA.2 of their internet appendix the case with price-impact costs, but they focus on the significance of the various characteristics for the mean-variance portfolio and do not study the capacity or profits of the strategies.

face intense competition. A distinguishing feature of our work is our focus on the effect of competition across *multiple* characteristics in the presence of trading diversification.

There is also a literature on crowding and tail risk. Brown, Howard, and Lundblad (2020) find that hedge fund exposures to a crowding factor explain tail risk. However, Barroso, Edelen, and Karehnke (2021) cast doubt on crowding as a stand-alone source of tail risk. These papers are related to the literature on the *dynamics* of market liquidity (Khandani and Lo, 2011; Nagel, 2012; Drechsler, Moreira, and Savov, 2020; Franzoni, Plazzi, and Cotelioglu, 2021). In contrast to this literature, and consistent with the capacity literature, we focus on the *static* effect of competition on price impact and profits from factor investing.

The remainder of this manuscript is organized as follows. Section 2 describes how we extend the parametric portfolios of Brandt, Santa-Clara, and Valkanov (2009) to allow for price-impact costs. Section 3 characterizes theoretically and empirically the trading-diversification mechanism for the case with a single investor, that is, ignoring competition. Section 4 uses a game-theoretic model with multiple investors to study the effect of competition on capacity and profits and calibrates the model using data on characteristics. Section 5 concludes. Appendix A uses an example to provide the intuition underlying trading diversification, Appendix B provides a more general analysis of the game-theoretic model, Appendix C uses mutual-fund data to provide empirical support for the main predictions of the game-theoretic model, Appendix D provides proofs for all results, and the Internet Appendix contains robustness checks.

2 Parametric portfolios with price-impact costs

We now explain how we extend the parametric portfolios of Brandt et al. (2009) to consider price-impact costs in factor investing.

2.1 Parametric portfolios

We consider a market with N stocks and K firm-specific characteristics. We standardize x_{kt} cross sectionally to have zero mean; that is, x_{kt} is a long-short portfolio, and thus, has zero cost. This is customary in cross-sectional asset pricing and it facilitates our analysis by removing the need for a budget constraint.[5] We also standardize the characteristic portfolio weight vectors so that the sum of the positive or negative weights is one; that is, a portfolio $\theta_k x_{kt}$ invests θ_k dollars in both the positive and negative legs. Finally, for the empirical analysis we consider *value-weighted* long-short characteristic portfolios to avoid allocating large weights to small firms that are difficult to trade.

Like Brandt et al. (2009), we consider a parametric portfolio policy such that the weight on a particular stock at time t is a linear function of *only* its weights in the K characteristic portfolios. Moreover, the same linear function is applied across stocks and over time.[6] Thus, the parametric portfolio at time t can be written as

$$w_t(\theta) = \sum_k x_{kt}\theta_k = X_t\theta,$$

where $\theta_k \in \mathbb{R}$ is the investment position in the kth characteristic, $\theta = (\theta_1, \theta_2, \dots, \theta_K)$ is the investment-position vector, and $X_t = (x_{1t}, x_{2t}, \dots, x_{Kt}) \in \mathbb{R}^{N\times K}$ is the matrix whose columns are the K long-short characteristic portfolios at time t. The return of the parametric portfolio at time $t+1$ is:[7]

$$r_{p,t+1} = w_t(\theta)^\top r_{t+1} = \sum_k \theta_k\, x_{kt}^\top r_{t+1} = \theta^\top X_t^\top r_{t+1}. \tag{1}$$

2.2 Price-impact costs

Firm characteristics may reflect risk or mispricing. If a characteristic reflects compensation for risk, then trading the characteristic will exert mainly temporary price impact, but if it reflects mispricing, then trading it may exert *both* temporary and permanent price impact. In

[5]There are both long-short and long-only factor-investing products. The advantages of long-short funds are that they are market neutral and they can exploit the favorable performance of the short leg. The main advantage of long-only products is that they do not require shorting, which may be costly. However, many long-short products can reduce costs by shorting the market index instead of individual stocks. For a systematic study of long-short mutual funds, see An, Huang, Lou, and Shi (2021).

[6]The linearity of the parametric portfolio policy is required for tractability of the game-theoretic model that we introduce in Section 4.1. Also, the assumption that the weights assigned to the characteristics are *constant* over time is consistent with the empirical literature on the capacity of quantitative strategies, which characterizes the largest investment position that can be allocated to a particular characteristic over the *entire* period before price-impact costs drive its net average return to zero; see Korajczyk and Sadka (2004), Ratcliffe et al. (2017), and Frazzini et al. (2018).

[7]Although r_{t+1} is a *payoff* because the parametric portfolio is a zero-cost long-short portfolio, for simplicity we refer to it as a *return*.

the main body of the manuscript, we consider the case where trading a characteristic exerts only temporary price impact, consistent with the empirical literature on the capacity of quantitative strategies (Korajczyk and Sadka, 2004; Lesmond, Schill, and Zhou, 2004; Novy-Marx and Velikov, 2016; Ratcliffe et al., 2017; Frazzini et al., 2018). However, Section IA.4 of the Internet Appendix shows that our findings are robust to considering *persistent* price-impact costs, which include as special cases permanent and temporary price-impact costs.

While several papers assume the price impact of a trade is linear in the amount traded (Korajczyk and Sadka, 2004; Novy-Marx and Velikov, 2016), empirical evidence finds that price impact grows with the square root of the amount traded (Torre and Ferrari, 1997; Grinold and Kahn, 2000; Almgren, Thum, Hauptmann, and Li, 2005; Ratcliffe et al., 2017; Frazzini et al., 2018). To capture either specification, we write the *price-impact function* at time t as:

$$\mathrm{PI}_t = \Lambda_t \ \mathrm{sign}(\Delta w_t) \circ |\Delta w_t|^{\alpha}, \tag{2}$$

where $\mathrm{sign}(x)$ is the componentwise sign of vector x, and the cases with linear and square-root price impact correspond to $\alpha = 1$ and $\alpha = 0.5$, respectively. The price-impact matrix

$$\Lambda_t = \mathrm{diag}(\lambda_{t1}, \lambda_{t2}, \dots, \lambda_{tN}) \in \mathbb{R}^{N\times N} \tag{3}$$

is the diagonal matrix whose nth element, λ_{tn}, is the price-impact parameter for the nth stock at time t, which is exogenous in our model; $\Delta w_t \in \mathbb{R}^N$ is the *aggregate-trade vector* at time t, defined as the vector that contains the net amount traded in the market for each stock aggregated across all investors and given by

$$\Delta w_t = \sum_k \theta_k \tilde{x}_{kt}, \qquad \text{where} \tag{4}$$

$$\tilde{x}_{kt} = x_{kt} - x_{k,t-1} \circ (e + r_t), \tag{5}$$

$x \circ y$ is the componentwise or Hadamard product of vectors x and y, and e is the N-dimensional vector of ones. The aggregate-trade vector can also be conveniently written in matrix notation as

$$\Delta w_t = \tilde{X}\theta, \tag{6}$$

where $\tilde{X}_t = (\tilde{x}_{1t}, \tilde{x}_{2t}, \dots, \tilde{x}_{Kt}) \in \mathbb{R}^{N\times K}$.

The *price-impact cost* at time t is the amount of trading multiplied by its price impact:

$$\text{PIC}_t = \Delta w_t^\top \, \text{PI}_t = \Delta w_t^\top \, \Lambda_t \, \text{sign}(\Delta w_t) \circ |\Delta w_t|^\alpha. \tag{7}$$

Substituting (6) into (7), the price-impact cost of rebalancing the portfolio at time t is

$$\text{PIC}_t = \theta^\top \tilde{X}_t^\top \, \Lambda_t \, \text{sign}(\tilde{X}_t \theta) \circ |\tilde{X}_t \theta|^\alpha. \tag{8}$$

2.3 Optimal parametric portfolio

The optimal portfolio at time t is given by the investment-position vector θ that maximizes the *conditional* expected portfolio return net of price-impact costs. However, a key insight of Brandt et al. (2009) is that the optimal parametric portfolio policy can be obtained by maximizing the *unconditional* expectation because the investment-position vector θ is assumed to be *constant* over time. In addition, for simplicity we assume that investors are risk neutral, although Section IA.1 in the Internet Appendix shows that our results are robust to considering risk-averse investors. Therefore, the optimal parametric portfolio is obtained by choosing the investment-position vector θ that maximizes the unconditional expectation of the difference between the portfolio return and the price-impact cost

$$\max_\theta \quad E\big[r_{p,t+1} - \text{PIC}_t\big], \tag{9}$$

in which $r_{p,t+1}$ and PIC_t are functions of θ, as specified in Equations (1) and (8), respectively.

3 Trading diversification with a single investor

To pave the way for our analysis of the impact of competition, we first characterize theoretically (in Section 3.1) and empirically (in Section 3.2) the trading-diversification mechanism for the case of a single investor, that is, ignoring competition. Then, in Section 4, we study the implications of trading diversification on the equilibrium among multiple competing investors.

3.1 Theoretical results

We first study theoretically how the price-impact cost of a single investor is reduced when she exploits a set of characteristics in combination (that is, accounting for trading diversification) compared to the case when she trades them in isolation (ignoring trading diversification).

Definition 3.1 Given K characteristics whose rebalancing trades follow a particular joint probability distribution, the *price-impact diversification ratio* for the nth stock is defined as the ratio of the unconditional expected price-impact cost required to rebalance the position on the nth stock for an equally weighted portfolio of the K characteristics (that is, accounting for trading diversification) to that required to rebalance the K characteristics in isolation (ignoring trading diversification). Mathematically,

$$\text{price-impact diversification ratio} := \frac{E\left[\lambda_{tn}\left|\sum_{k=1}^{K}\tilde{x}_{ktn}\right|^{1+\alpha}\right]}{\sum_{k=1}^{K}E\left[\lambda_{tn}\left|\tilde{x}_{ktn}\right|^{1+\alpha}\right]}, \tag{10}$$

where λ_{tn} is the nth stock price-impact parameter at time t, that is, the nth element of the diagonal matrix Λ_t in (3), and $\tilde{x}_{ktn}$ is the trade on the nth stock required to rebalance the kth characteristic at time t, that is, the nth element of vector $\tilde{x}_{kt}$ in (5).

Note that a price-impact diversification ratio smaller than one implies that there is a reduction in price-impact costs from combining characteristics. For instance, a price-impact diversification ratio of 0.75 indicates that the price-impact cost of trading the characteristics in combination is 25% smaller than that of trading them in isolation. On the other hand, a price-impact diversification ratio of one implies that there are no diversification benefits from combining the characteristics. Finally, a price-impact diversification ratio larger than one implies that the price-impact cost of trading the characteristics in combination is higher than that of trading them in isolation.[8]

The following proposition characterizes analytically the price-impact diversification ratio for $\alpha > -1$. DeMiguel et al. (2020, Proposition 3) characterize this ratio for the case with *proportional transaction costs*, $\alpha = 0$. Here, we generalize their result to the case with $\alpha > -1$, which includes the two most important *price-impact-cost models*: (i) $\alpha = 1$, implying a linear price-impact function, and thus, quadratic price-impact costs and (ii) $\alpha = 0.5$, implying a square-root price-impact function, and thus, subquadratic price-impact costs.

Proposition 3.1 *Assume that the trades in the nth stock required to rebalance K characteristics, that is, the quantities $\tilde{x}_{ktn}$ for $k = 1, 2, \ldots, K$, are jointly distributed as a Normal*

[8]To see how the price-impact diversification ratio could be larger than one, note that if the rebalancing trades of K characteristics are highly positively correlated, then the price-impact cost of trading them in isolation could be smaller than that of trading them in combination because price-impact costs are a strictly convex function of the amount traded.

distribution with zero mean and positive definite covariance matrix Ω*. Moreover, assume the* n*th stock price-impact parameter is independently distributed from the rebalancing trades. Then, for any* $\alpha > -1$ *the*

$$\text{price-impact diversification ratio} = \frac{\left(\sum_{k=1}^{K} \sigma_k^2 + \sum_{k=1}^{K} \sum_{l \neq k} \rho_{kl} \sigma_k \sigma_l\right)^{\frac{1+\alpha}{2}}}{\sum_{k=1}^{K} \sigma_k^{1+\alpha}},$$

where σ_k^2 *is the variance of the rebalancing trade* $\tilde{x}_{ktn}$ *and* ρ_{kl} *is the correlation between the rebalancing trades* $\tilde{x}_{ktn}$ *and* $\tilde{x}_{ltn}$*. If, in addition, the covariance matrix* Ω *is symmetric with respect to the* K *characteristics, that is, if* $\sigma_k^2 = \sigma^2$ *for all* k *and* $\rho_{kl} = \rho$ *for all* $k \neq l$*, then*[9]

$$\text{price-impact diversification ratio} = \frac{\left[K(1 + (K-1)\rho)\right]^{\frac{1+\alpha}{2}}}{K} \tag{11}$$

and the price-impact diversification ratio is strictly smaller than one if and only if

$$\rho < \bar{\rho} = \frac{K^{\frac{1-\alpha}{1+\alpha}} - 1}{K - 1}. \tag{12}$$

We now discuss Proposition 3.1 focusing, for simplicity, on the symmetric case with $\sigma_k^2 = \sigma^2$ for all k and $\rho_{kl} = \rho$ for all $k \neq l$. Equation (12) shows that the threshold correlation $\bar{\rho}$ below which the price-impact diversification ratio is smaller than one depends on the form of the price impact given by α and the number of characteristics combined K. For the case with linear price impact, $\alpha = 1$, we have that $\bar{\rho} = 0$, and thus, combining characteristics reduces price-impact costs only if their portfolio-rebalancing trades are negatively correlated.

However, for the empirically relevant case of square-root price impact, $\alpha = 0.5$, Proposition 3.1 shows that the price-impact diversification ratio can be smaller than one even if the rebalancing trades are positively correlated, as long as the correlation is below the threshold $\bar{\rho} = (K^{1/3} - 1)/(K - 1) > 0$. That is, with square-root price impact, combining characteristics leads to a reduction of price-impact costs *even if* the rebalancing trades of the characteristics are moderately *positively* correlated.[10] This difference is driven by the

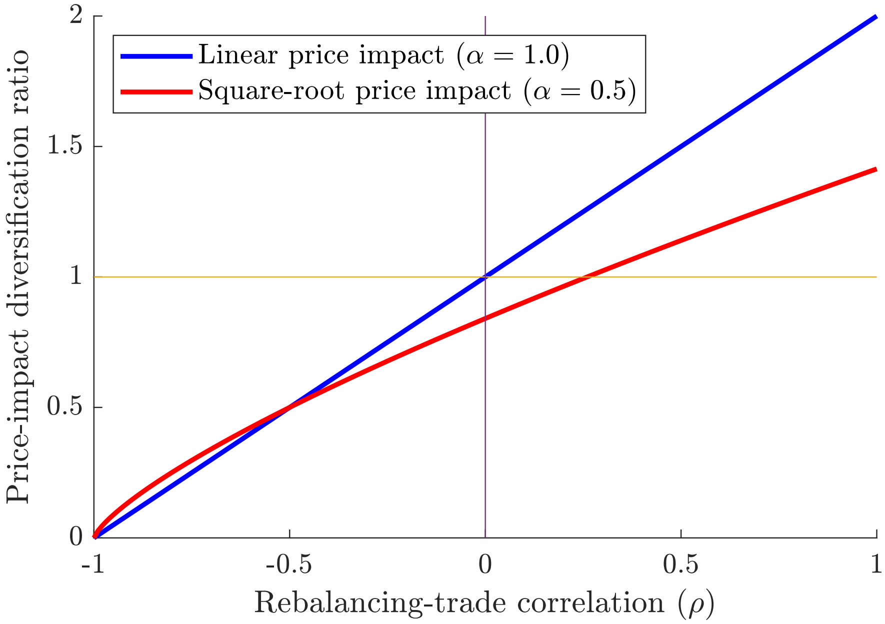
[9]Note that the term $K(1 + (K-1)\rho)$ is strictly positive because Ω is positive definite.

[10]Proposition 3.1 shows that *trading diversification* is fundamentally different from variance-risk diversification, which takes place whenever asset returns are not *perfectly* correlated. For the case with linear price impact, trading diversification occurs *only for strictly negative correlation* between the rebalancing trades of different characteristics. Even for the case with square-root price impact, combining characteristics reduces price-impact costs only if the correlation between rebalancing trades is below a positive threshold that is strictly smaller than one.

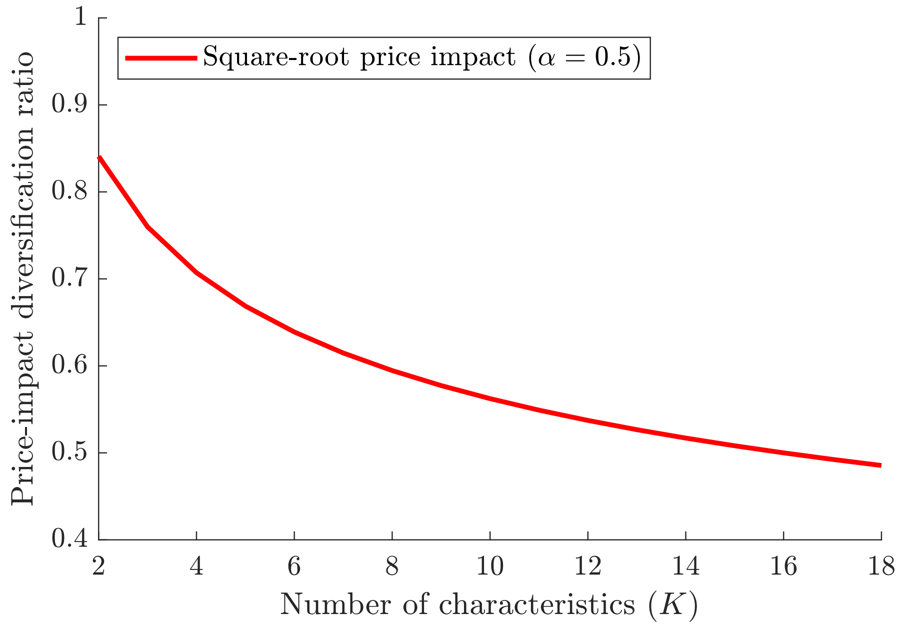
Figure 2: Price-impact diversification ratio: Theoretical results

This figure depicts the price-impact diversification ratio given in (10) for characteristics satisfying the assumptions of Proposition 3.1 and for the case where the covariance matrix of rebalancing trades is symmetric with respect to the characteristics; that is, the case where $\sigma_k^2 = \sigma^2$ for all k and $\rho_{kl} = \rho$ for all $k \neq l$. Panel (a) considers the case with two characteristics and shows the price-impact diversification ratio on the vertical axis as a function of the correlation between the rebalancing trades of the two characteristics on the horizontal axis. The two curves correspond to the cases with square-root price-impact function (subquadratic price-impact costs, $\alpha = 0.5$) and linear price-impact function (quadratic price-impact costs, $\alpha = 1$). Panel (b) shows the price-impact diversification ratio on the vertical axis as a function of the number of characteristics combined for the case with $\rho = 0$ and square-root price-impact function ($\alpha = 0.5$).

(a) Effect of rebalancing trade correlation for two characteristics



(b) Effect of number of characteristics for zero rebalancing-trade correlation



fact that the square-root price-impact function ($\alpha = 0.5$) assigns a much lower weight to large trades than the linear price-impact function. Appendix A uses a simple example to provide intuition for the difference in the theoretical results for the cases with linear and square-root price impact.

These theoretical findings are illustrated in Figure 2. Panel (a) depicts how the price-impact diversification ratio depends on the rebalancing trade correlation ρ for the case with two characteristics and linear ($\alpha = 1$) or square-root price impact ($\alpha = 0.5$). The graph shows that for the case with square-root price impact ($\alpha = 0.5$) the ratio is smaller than one provided $\rho < \bar{\rho} = 2^{1/3} - 1 \approx 0.26$. For example, if $\rho = 0$, the price-impact diversification ratio for the case with two characteristics is around 0.84, indicating that the price-impact cost of trading the two characteristics in combination is around 16% smaller than the average cost of trading them in isolation. Panel (b) illustrates the effect of the number of characteristics K on the price-impact diversification ratio for the case with zero rebalancing-trade correlation ($\rho = 0$) and square-root price impact ($\alpha = 0.5$). The plot shows that the benefits from trading diversification increase substantially with the number of characteristics combined and the price-impact diversification ratio ranges from 84% for the case with two characteristics to around 50% for the case with $K = 18$ characteristics.

In this section, we have shown that, with a square-root price impact, combining characteristics leads to a reduction in price-impact costs even when the rebalancing trades of the characteristics are uncorrelated or moderately positively correlated. In the next section, we examine the magnitude of this effect empirically for the case with a single investor exploiting 18 characteristics.

3.2 Empirical results

To evaluate the magnitude of the trading-diversification benefits for a single investor from combining characteristics, we require a historical sample of the rebalancing-trade vectors, $\tilde{x}_{kt}$, the characteristic-portfolio returns, $x_{kt}^{\top} r_{t+1}$, and an estimate of the price-impact cost for each stock.

To obtain a historical sample for the rebalancing-trade vectors and characteristic-portfolio returns, we compile data on stock returns as well as for the 18 characteristics

Table 1: List of characteristics considered

This table lists the 18 characteristics we consider, ordered alphabetically by acronym. The first column gives the number of the characteristic, the second column gives the characteristic's definition, the third column gives the acronym, and the fourth and fifth columns give the authors who analyzed them, and the date and journal of publication.

#	Characteristic and definition	Acronym	Author(s)	Date, Journal
1	Asset growth: Annual percent change in total assets	agr	Cooper, Gulen & Schill	2008, JF
2	Beta: Estimated market beta from weekly returns and equal weighted market returns for 3 years ending month $t-1$ with at least 52 weeks of returns	beta	Fama & MacBeth	1973, JPE
3	Book to market: Book value of equity divided by end of fiscal-year market capitalization	bm	Rosenberg, Reid & Lanstein	1985, JPM
4	Industry adjusted book to market: Industry adjusted book-to-market ratio	bm_ia	Asness, Porter & Stevens	2000, WP
5	Industry adjusted change in asset turnover: 2-digit SIC fiscal-year mean adjusted change in sales divided by average total assets	chatoia	Soliman	2008, TAR
6	Change in tax expense: Percent change in total taxes from quarter $t-4$ to t	chtx	Thomas & Zhang	2011 JAR
7	Gross profitability: Revenues minus cost of goods sold divided by lagged total assets	gma	Novy-Marx	2013 JFE
8	Industry sales concentration: Sum of squared percent of sales in industry for each company	herf	Hou & Robinson	2006, JF
9	12-month momentum: 11-month cumulative returns ending one month before month-end	mom12m	Jegadeesh	1990, JF
10	1-month momentum: 1-month cumulative return	mom1m	Jegadeesh	1990, JF
11	Market capitalization: Natural log of market capitalization at end of month $t-1$	mve	Banz	1981, JFE
12	Δ% gross margin - Δ% sales: Percent change in gross margin minus percent change in sales	pchgm_pchsale	Abarbanell & Bushee	1998, TAR
13	Financial-statements score: Sum of 9 indicator variables to form fundamental health score	ps	Piotroski	2000, JAR
14	R&D to market cap: R&D expense divided by end-of-fiscal-year market capitalization	rd_mve	Guo, Lev & Shi	2006, JBFA
15	Return volatility: Standard deviation of daily returns from month $t-1$	retvol	Ang, Hodrick, Xing & Zhang	2006, JF
16	Volatility of share turnover: Monthly standard deviation of daily share turnover	std_turn	Chordia, Subrahmanyan & Anshuman	2001, JFE
17	Unexpected quarterly earnings: Unexpected quarterly earnings divided by fiscal-quarter-end market cap. Unexpected earnings is I/B/E/S actual earnings minus median forecasted earnings if available, else it is the seasonally differenced quarterly earnings before extraordinary items from Compustat quarterly file	sue	Rendelman, Jones & Latane	1982, JFE
18	Zero trading days: Turnover weighted number of zero trading days for most recent month	zerotrade	Liu	2006, JFE

listed in Table 1, which include the traditional characteristics size, value, and momentum plus the 15 characteristics that DeMiguel et al. (2020) find jointly significant for explaining the cross section of stock returns. We combine U.S. stock-market information from CRSP, Compustat, and I/B/E/S from January 1980 to December 2018.[11] Our database contains every firm traded on the NYSE, AMEX, and NASDAQ exchanges. We then remove firms with negative book-to-market ratios. As in Brandt et al. (2009), we also remove firms below the 20th percentile of market capitalization because these are very small firms that are difficult to trade.

We form value-weighted long-short portfolios for each characteristic by going long on stocks with values of the characteristic above the 70th percentile and going short stocks with values of the characteristic below the 30th percentile. We standardize the value-weights so that both the positive and negative weights sum to one for each characteristic. Then, we

[11]We thank Jeremiah Green for sharing the code to download the data in Green, Hand, and Zhang (2017).

use (5) to compute the monthly rebalancing-trade vectors and a single-characteristic version of (1) to compute the returns of each characteristic portfolio.

To estimate the price-impact cost for the nth stock, we use the results of Frazzini et al. (2018), who use a trade-execution dataset from a large money manager covering a 19-year period to estimate the following panel regression for the price impact of a trade on the nth stock

$$\mathrm{PI}_{tn} = a_{tn} + b\,\mathrm{vr}_{tn} + c\ \mathrm{sign}(\mathrm{vr}_{tn})\sqrt{|\mathrm{vr}_{tn}|}, \tag{13}$$

where $\mathrm{vr}_{tn} = 100 \times \Delta w_{tn}/\mathrm{dtv}_{tn}$ is the signed dollar value of a trade Δw_{tn} as a percentage of the stock's average daily dollar volume dtv_{tn}. The second and third terms on the right-hand side of (13) account for the linear and square-root price impact of trading, respectively. The first term a_{tn} captures the effect of explanatory variables that do not depend on the trade size such as a time trend, the stock's log market capitalization and idiosyncratic volatility, and the monthly variance of the CRSP value-weighted index. The panel regression in (13) is a generalization of the price-impact function given in Equation (2) because it allows for additional explanatory variables collected in a_{tn} and a term linear in vr_{tn}, in addition to a square-root term. Frazzini et al. (2018) find that the coefficient c in (13) is highly statistically significant, whereas the coefficient b is not significant, consistent with the findings of Torre and Ferrari (1997), Grinold and Kahn (2000), Almgren et al. (2005), and Ratcliffe et al. (2017). We rely on the estimates of a_{tn}, b, and c reported in Column (9) of Table VII in Frazzini et al. (2018) to characterize the price-impact cost of a trade in the nth stock.

Based on the historical sample of rebalancing-trade vectors and the stock price-impact model of Frazzini et al. (2018), Table 2 compares the capacity, optimal investment, and optimal annual profit of a single investor that exploits the 18 characteristics when considered in isolation (without trading diversification) and in combination (with trading diversification). We obtain the optimal investment and profit by solving the parametric portfolio problem (9) for each of the 18 characteristics in isolation and in combination.[12] The investment is given by the optimal value of θ and the *annual* profit is 12 times the optimal objective of problem (9). We obtain the capacity of each characteristic in isolation by computing the maximum invest-

[12] For the price-impact cost model of Frazzini et al. (2018), which contains both linear and square-root price-impact terms, there are no closed-form expressions for the optimal investment and profit, so we compute these numerically. Also, as explained in Section 2.1, the investment position θ_i represents the dollars invested on the long leg and on the short leg of the ith characteristic portfolio.

Table 2: Impact of trading diversification on capacity, investment, and profit

This table reports the capacity, optimal investment, and optimal annual profit of a single investor that exploits the 18 characteristics ignoring trading diversification (in isolation) and accounting for trading diversification (in combination). For each characteristic, the first column reports its acronym and the remaining columns report its capacity, optimal investment, and optimal profit when considered in isolation and in combination, as well as the percentage increase in these quantities when the characteristic is considered in combination instead of in isolation. We obtain the optimal investment and profit by solving problem (9) for each of the 18 characteristics in isolation and in combination, with the price-impact cost PIC_t evaluated using the model of Frazzini et al. (2018) in Equation (13). The investment is given by the optimal value of θ and the annual profit is 12 times the optimal objective of problem (9). We express all quantities in terms of market capitalization at the end of our sample (December 2018).

	Capacity			Investment			Profit		
Characteristic	Isol. ($bill.)	Comb. ($bill.)	Incr. (%)	Isol. ($bill.)	Comb. ($bill.)	Incr. (%)	Isol. ($mill.)	Comb. ($mill.)	Incr. (%)
gma	105.502	133.764	27	52.751	64.106	22	324.52	443.48	37
rd_mve	66.163	66.009	−0	31.954	31.635	−1	918.93	923.85	1
bm	32.510	41.174	27	15.346	19.733	29	129.17	178.12	38
herf	6.389	40.488	534	2.832	19.404	585	2.18	23.29	970
agr	16.082	25.681	60	7.528	12.308	63	69.73	133.93	92
ps	4.819	7.542	57	2.223	3.615	63	11.36	24.76	118
chatoia	1.654	6.392	287	0.753	3.064	307	2.00	12.18	510
beta	1.319	5.952	351	0.601	2.853	374	0.94	7.18	667
bm_ia	0.000	5.572	-	0.000	2.670	-	0.00	0.77	-
mom12m	1.007	3.090	207	0.456	1.481	225	3.19	15.87	398
chtx	0.768	2.348	206	0.351	1.125	220	1.45	7.44	414
sue	0.869	2.265	161	0.397	1.085	173	2.41	9.19	281
retvol	0.201	1.621	708	0.088	0.777	785	0.50	9.40	1790
pchgm_pchsale	1.450	1.068	−26	0.658	0.512	−22	1.63	2.55	57
std_turn	0.001	1.008	94076	0.000	0.483	-	0.00	1.62	-
mom1m	0.002	0.552	31145	0.000	0.265	-	0.00	2.86	-
zerotrade	0.005	0.424	8965	0.002	0.203	9865	0.00	1.47	78499
mve	0.000	0.125	-	0.000	0.060	-	0.00	0.02	-
Total	238.739	345.076	45	115.941	165.378	43	1467.99	1797.98	22

ment that can be allocated to each characteristic before price-impact costs erode any profits. We obtain the capacity of the 18 characteristics in combination by scaling up the optimal parametric-portfolio vector for the case where the 18 characteristics are exploited in combination until price-impact costs erode any profits. As in Korajczyk and Sadka (2004) and Novy-Marx and Velikov (2016), we express all quantities in terms of market capitalization at the end of our sample (December 2018).

The capacity estimates in Table 2 are consistent with those in the literature; for instance, the total capacity aggregated across the 18 characteristics when considered in isolation is around $239 billion. When considered in isolation, the "gross profitability (gma)" characteristic has the largest capacity of around $105 billion. Other popular characteristics such as "R&D to market capitalization (rd_mve)," "value (book to market, bm)," and "investment (asset growth, agr)" have capacities in isolation of more than $10 billion each. In unreported results, we also find that the median trade size of the single investor over time and across stocks is $505,827, which is of the same order of magnitude as that reported in Table I of Frazzini et al. (2018) for their trade execution database, $356,900. This indicates that it is reasonable to use the price-impact cost model of Frazzini et al. (2018) in our context.

More importantly, Table 2 shows that trading diversification has a substantial effect on capacity, investment, and profits. The last row of the table (Total) shows that exploiting the 18 characteristics in combination leads to a 45% increase in total capacity from 239 to $345 billion, a 43% increase in total optimal investment from 116 to $165 billion, and a 22% increase in total annual profits from 1.5 to $1.8 billion.[13]

Figure 3 reports the empirical price-impact diversification ratio as a function of the number of characteristics considered, when we invest in each characteristic the amount that is optimal for a single investor that considers all 18 characteristics in combination; that is, the amount in the sixth column of Table 2.[14] For each number of characteristics $K = 1, 2, \ldots, 18$ (depicted on the horizontal axis), we consider all combinations of the 18 characteristics taken in groups of K and report the mean along with the 5th and 95th percentiles of price-impact diversification ratio across the combinations.

The figure shows that the benefits from trading diversification increase substantially with the number of characteristics, and the average price-impact diversification ratio is around 84% for the case where all 18 characteristics are considered; that is, there is a reduction of around 16% in price-impact costs when combining all characteristics using their optimal investment weights compared to considering them in isolation.[15] Moreover, the

[13]Moreover, Section IA.2 of the Internet Appendix reports the result of a subsample analysis and shows that trading diversification has a substantial effect in both the first and second halves of our sample.

[14]We have reproduced Figure 3 for the case where the characteristics are equally weighted and the results are very similar, with a price-impact cost reduction of around 18% from combining 18 characteristics, compared to exploiting them in isolation.

[15]Although substantial, this price-impact cost reduction is smaller than that predicted by Proposition 3.1 because the multivariate distribution of the *empirical* rebalancing trades is neither symmetric nor normally distributed.

Figure 3: Price-impact diversification ratio: Empirical results

This figure depicts the empirical price-impact diversification ratio as a function of the number of characteristics combined, when we invest in each characteristic the amount that is optimal when all 18 characteristics are considered in combination. For each number of characteristics K (depicted on the horizontal axis), we consider all combinations of the 18 characteristics taken in groups of K and report the mean (solid red line) and the 5th and 95th percentiles (dashed black lines) of price-impact diversification ratio across the combinations.

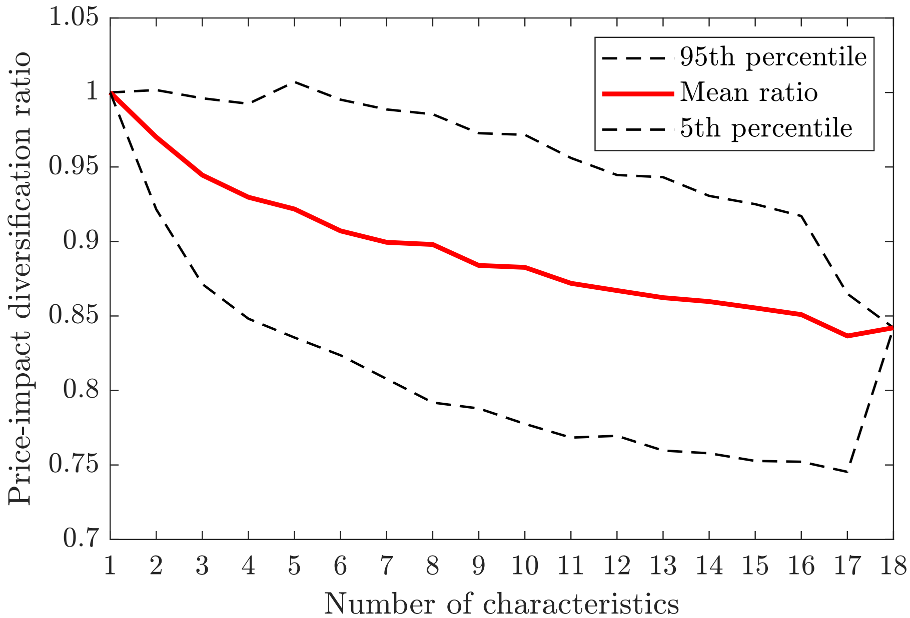


figure shows that the results in Table 2 are robust to considering different subsets of characteristics. To see this, note that we do not re-optimize the weights of the characteristics when considering $K < 18$, but rather use the weights that are optimal when combining all 18 characteristics, yet there are substantial benefits in terms of price-impact cost from combining characteristics.

In this section, we have shown that trading diversification has a substantial effect on the optimal investment positions, price-impact costs, and profits of a single investor. In the next section, we study theoretically how these quantities are affected by competition among multiple investors.

4 Competition with trading diversification

We now study how the trading-diversification mechanism characterized in the previous section affects capacity, investment, price-impact costs, and profits when two groups of investors compete to exploit different characteristics. Section 4.1 proposes a game-theoretic model of competition and characterizes its equilibrium in closed form, Sections 4.2–4.4 discuss the effect of competition in the presence of trading diversification, and Section 4.5 calibrates the game-theoretic model using empirical data to gauge the magnitude of this effect.

4.1 Game-theoretic model of competition

We now extend the parametric-portfolio framework introduced in Section 2 to develop a game-theoretic model of competition. For simplicity, in the theoretical model we consider two groups of investors exploiting two different characteristics, as illustrated in Figure 4. The first group contains I_1 investors exploiting the first characteristic and the second group contains I_2 investors exploiting the second characteristic. As the characteristics of the N stocks change over time, each investor needs to rebalance her portfolio monthly so that it has the correct loading on each characteristic. There is a negative externality among investors in each group, because when the ith investor in the kth characteristic increases her investment position, the rest of the investors exploiting the kth characteristic incur *higher* price-impact costs. There is also a positive externality across the two groups of investors because when investors in one group increase their investment position, the investors in the other group incur *lower* price-impact costs.

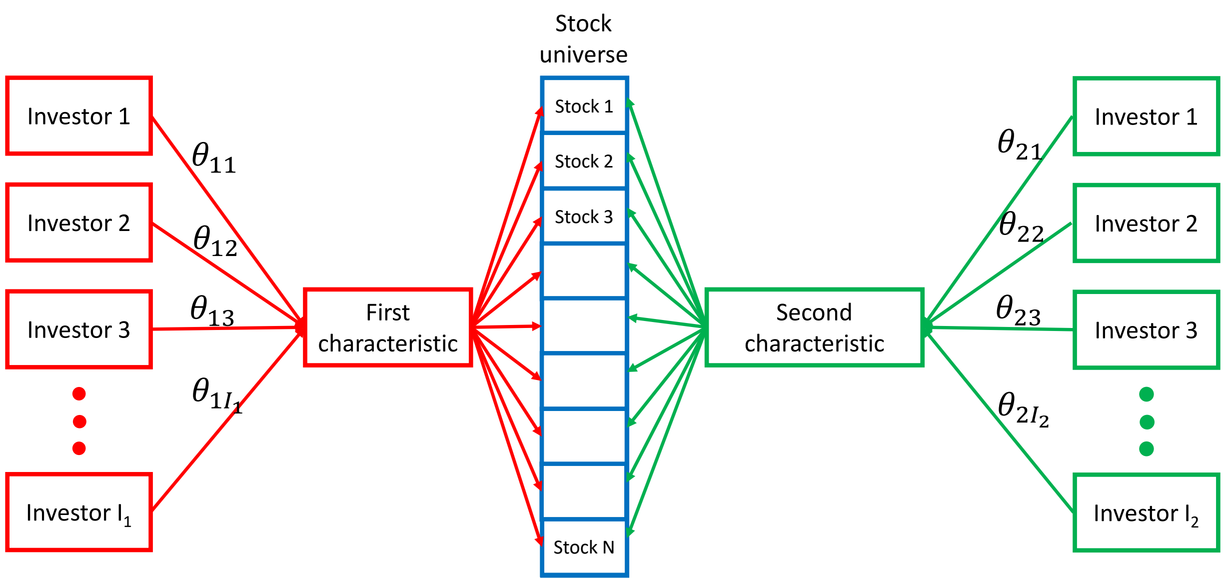
Our theoretical analysis relies on the following assumption.

Assumption 4.1 *The following three conditions hold: (1) aggregate trading has a linear impact on prices; (2) there is a strictly positive probability that the rebalancing-trade vectors of the two characteristics, $\tilde{x}_{1t}$ and $\tilde{x}_{2t}$, are nonzero and that they are not perfectly collinear; and (3) the average return of the second characteristic is zero.*

Assumption 4.1(1) is required for tractability, Assumption 4.1(2) rules out the unrealistic case where the characteristics are time invariant or perfectly collinear, and we make Assumption 4.1(3) to simplify the exposition. Appendix B generalizes the theoretical results for the case where Assumption 4.1(3) does not hold. Moreover, we relax all three assumptions in the empirical calibration of the game-theoretic model in Section 4.5.

Figure 4: Game-theoretic model of competition

This figure illustrates the game-theoretic model of competition in factor investing. There are two groups of investors exploiting two different characteristics. The first group contains I_1 investors exploiting the first characteristic (in red) and the second group contains I_2 investors exploiting the second characteristic (in green). The investment position of the ith investor exploiting the kth characteristic is θ_{ki}.



Consistent with the parametric portfolios introduced in Section 2, each investor in the game maximizes the expectation of the difference between her portfolio return and price-impact cost. Then, the following proposition shows that the decision problem of each investor can be written as a tractable optimization problem.

Proposition 4.1 *Let Assumption 4.1 hold. Then, the decision problems of the ith investors in the first and second characteristics are*

$$\max_{\theta_{1i}} \quad \underbrace{\theta_{1i}\mu_1}_{\textit{mean return}} \underbrace{- \theta_{1i}\lambda_1\theta_{1i} - \overbrace{\theta_{1i}\lambda_1\textstyle\sum_{j\neq i}\theta_{1j}}^{\textit{negative externality}} - \overbrace{\theta_{1i}\lambda_{12}\textstyle\sum_{j=1}^{I_2}\theta_{2j}}^{\textit{positive externality}}}_{\textit{price-impact cost}}, \quad \textit{and} \tag{14}$$

$$\max_{\theta_{2i}} \quad \underbrace{- \theta_{2i}\lambda_2\theta_{2i} - \overbrace{\theta_{2i}\lambda_2\textstyle\sum_{j\neq i}\theta_{2j}}^{\textit{negative externality}} - \overbrace{\theta_{2i}\lambda_{12}\textstyle\sum_{j=1}^{I_1}\theta_{1j}}^{\textit{positive externality}}}_{\textit{price-impact cost}}, \quad \textit{respectively,} \tag{15}$$

where θ_{ki} is the investment position of the ith investor exploiting the kth characteristic, $\mu_1 = E[x_{1t}^\top r_{t+1}]$ is the average return of the first characteristic portfolio, $\lambda_k = E[\tilde{x}_{kt}^\top \Lambda_t \tilde{x}_{kt}]$ is *the price-impact parameter for the kth characteristic, $\lambda_{12} = E[\tilde{x}_{1t}^\top \Lambda_t \tilde{x}_{2t}]$ is the price-impact parameter for the interaction between the two characteristics, Λ_t is the price-impact matrix defined in* (3)*, and $\tilde{x}_{kt}$ is the rebalancing-trade vector for the kth characteristic in* (5)*.*

In Equation (14), the first term in the objective function of the ith investor exploiting the first characteristic is her mean return ($\theta_{1i}\mu_1$) and the remaining three terms capture her price-impact costs. In particular, the second term is the price-impact cost that the investor incurs due solely to her own trading activity ($\theta_{1i}\lambda_1\theta_{1i}$), the third term is the price-impact cost that the investor incurs due to the negative externality from the trading activity of other investors exploiting the first characteristic ($\theta_{1i}\lambda_1\sum_{j\neq i}\theta_{1j}$), and the fourth term is the price-impact cost *reduction* that the investor enjoys due to the positive externality from the trading activity of investors exploiting the second characteristic ($\theta_{1i}\lambda_{12}\sum_{j=1}^{I_2}\theta_{2j}$).[16]

The utility of the ith investor exploiting the second characteristic, in Equation (15), is similar to that of the ith investor in the first characteristic, except that for simplicity we assume the average return of the second characteristic is zero, as stated in Assumption 4.1(3).

The following proposition provides closed-form expressions for the equilibrium quantities in the game-theoretic model. We employ the subscript "d" to denote quantities in a *decentralized* setting with multiple competing investors, which we will later contrast with the centralized setting.

Proposition 4.2 *Let Assumption 4.1 hold. Then, there exists a unique Nash equilibrium that is symmetric with respect to the I_1 investors exploiting the first characteristic and with respect to the I_2 investors exploiting the second. Moreover, the investment positions of the ith investors exploiting the first and second characteristics are*

$$\theta_{1id} = \frac{(I_2+1)\,\lambda_2\mu_1}{(I_1+1)(I_2+1)\lambda_1\lambda_2 - I_1 I_2 \lambda_{12}^2} \qquad \textit{and} \tag{16}$$

$$\theta_{2id} = -\frac{I_1\lambda_{12}\mu_1}{(I_1+1)(I_2+1)\lambda_1\lambda_2 - I_1 I_2 \lambda_{12}^2}, \quad \textit{respectively,} \tag{17}$$

[16] We exclude from our analysis the trivial case with $\lambda_{12} = 0$, in which the decisions of the two groups of investors are independent. Under Assumption 4.1(3), we can assume without loss of generality that $\lambda_{12} > 0$ because it is straightforward to show that for the case where the second characteristic has zero mean return, changing the sign of the second characteristic does not alter the equilibrium. Also, Proposition 4.5 shows that $\theta_{2i} < 0$ in equilibrium, and thus the fourth term in Equation (14) does indeed lead to a reduction in price-impact costs.

the profit of the ith investor exploiting the kth characteristic is

$$\pi_{kid} = \lambda_k \theta_{kid}^2, \tag{18}$$

the price-impact cost of the ith investor exploiting the kth characteristic is

$$\zeta_{kid} = \mu_k \theta_{kid} - \pi_{kid} = \mu_k \theta_{kid} - \lambda_k \theta_{kid}^2 \tag{19}$$

and the price impact experienced by the ith investor exploiting the kth characteristic is

$$\xi_{kid} = \frac{\zeta_{kid}}{\theta_{kid}} = \mu_k - \lambda_k \theta_{kid}. \tag{20}$$

In Sections 4.2–4.4 below, we use these closed-form expressions to understand the impact of competition in the presence of trading diversification on the equilibrium. Our discussion parallels that of Figure 1 in the introduction. To study the effect of crowding, we start by considering the case where there are only investors exploiting the first characteristic ($I_1 \geq 1,\ I_2 = 0$). Then, to characterize how competition among investors exploiting the second characteristic alleviates crowding in the first characteristic, we consider the case where there are investors exploiting both characteristics ($I_1 \geq 1$ and $I_2 \geq 1$). Finally, we consider the centralized setting where a single investor exploits both characteristics.

4.2 Competition in the first characteristic

To set the stage for our main insight about trading diversification, we begin by considering the case where there are only investors exploiting the first characteristic ($I_1 \geq 1$ and $I_2 = 0$). We start by identifying the *capacity* of the first characteristic, defined as the aggregate investment position for which aggregate profits become zero. From (14), we have that the aggregate profits for the case where there are no investors exploiting the second characteristic are $\pi_1 = \theta_1 \mu_1 - \theta_1 \lambda_1 \theta_1$, where the aggregate investment position in the first characteristic is $\theta_1 = \sum_{i=1}^{I_1} \theta_{1i}$. Thus, the capacity of the first characteristic is $C(I_2 = 0) = \mu_1 / \lambda_1$. Then, the following proposition characterizes the equilibrium investment positions, profits, and price-impact costs.

Proposition 4.3 *Let Assumption 4.1 hold and consider the case where there are only investors exploiting the first characteristic ($I_1 \geq 1$ and $I_2 = 0$). Then, there exists a unique*

Nash equilibrium, which is symmetric across the I_1 investors. Moreover, the aggregate investment position of the investors in the first characteristic is

$$\theta_{1d} = I_1\theta_{1id} = \frac{I_1}{I_1+1}\frac{\mu_1}{\lambda_1}, \tag{21}$$

and their aggregate profit and price-impact cost are

$$\pi_{1d} = I_1\pi_{1id} = I_1\lambda_1\theta_{1id}^2 = \frac{I_1}{(I_1+1)^2}\frac{\mu_1^2}{\lambda_1}, \quad \text{and} \tag{22}$$

$$\zeta_{1d} = I_1\zeta_{1id} = I_1^2\lambda_1\theta_{1id}^2 = \frac{I_1^2}{(I_1+1)^2}\frac{\mu_1^2}{\lambda_1}, \quad \text{respectively.} \tag{23}$$

Furthermore, the following monotonicity properties hold:

1. *The aggregate investment position $\theta_{1d} = I_1\theta_{1id}$ is increasing in I_1 and converges to the strategy's capacity μ_1/λ_1 as $I_1 \to \infty$.*
2. *The aggregate profits $\pi_{1d} = I_1\pi_{1id}$ are decreasing in I_1 and converge to zero as $I_1 \to \infty$.*
3. *The aggregate price-impact costs $\zeta_{1d} = I_1\zeta_{1id}$ are increasing in I_1 and converge to μ_1^2/λ_1 as $I_1 \to \infty$.*

The intuition underlying Proposition 4.3, which is shown in Panel (a) of Figure 1, is as follows. A single investor maximizes her profits by investing *half* of the capacity, $\theta_{1d} = \mu_1/2\lambda_1 = C(I_2 = 0)/2$. Note that this is the first-best allocation that maximizes aggregate profits in the absence of a second characteristic because the single investor acts as a monopolist exploiting the first characteristic. However, when multiple investors compete to exploit the first characteristic, there is a negative externality among them because they do not internalize in their objective function how their investment decisions affect each other's price-impact costs. Consequently, as the number of investors I_1 increases, their aggregate investment position increases and their aggregate profit decreases because their aggregate price-impact cost increases faster than their aggregate gross return. In the limit, as the number of investors goes to infinity, the externality pushes them to overinvest until price-impact costs completely erode any profits from trading the first characteristic, a result that parallels the seminal finding of Cournot (1838). Thus, Proposition 4.3 establishes the base-case result that *competition among investors exploiting the same characteristic erodes their profits because of the negative externality of their price impact on each other.*

4.3 Competition across characteristics

To study the effect of competition across different characteristics, we now consider the case where there may also be investors exploiting the second characteristic ($I_2 \geq 0$). We first characterize how competition among investors exploiting the second characteristic increases the capacity of the first characteristic.

Proposition 4.4 *Under Assumption 4.1, the capacity of the first characteristic is*

$$C(I_2) = \frac{\mu_1}{\lambda_1 - \frac{I_2}{I_2+1}\frac{\lambda_{12}^2}{\lambda_2}},$$

where λ_2 is the price-impact cost parameter for the second characteristic. Moreover, $C(I_2)$ is monotonically increasing in I_2 for any $I_2 \geq 0$.

We then characterize how the aggregate investment position, profits, and price impact for the first characteristic change with the number of investors exploiting the second characteristic, I_2.

Proposition 4.5 *Let Assumption 4.1 hold and $I_1 < \infty$. Then, the equilibrium quantities in the decentralized setting satisfy the following conditions with respect to the number of investors exploiting the second characteristic, $I_2 \geq 0$:*

1. *The aggregate investment position in the first characteristic $\theta_{1d} = I_1\theta_{1id}$ is strictly positive and increasing in I_2.*
2. *The aggregate profit from the first characteristic $\pi_{1d} = I_1\pi_{1id}$ is strictly positive and increasing in I_2.*
3. *The price impact from exploiting the first characteristic, $\xi_{1id} = \lambda_1 I_1\theta_{1id} + \lambda_{12}I_2\theta_{2id}$, is decreasing in I_2.*
4. *For $I_2 \geq 1$, the investment position in the second characteristic $\theta_{2d} = I_2\theta_{2id}$ is strictly negative and decreasing in I_2; that is, it is increasing in absolute value.*
5. *For $I_2 \geq 1$, the aggregate profit from the second characteristic $\pi_{2d} = I_2\pi_{2id}$ is strictly positive and decreasing in I_2 provided that $I_2 > (I_1+1)\lambda_1\lambda_2/[(I_1+1)\lambda_1\lambda_2 - I_1\lambda_{12}^2]$ and converges to zero as $I_2 \to \infty$.*

We now discuss the intuition underlying Propositions 4.4 and 4.5; our discussion parallels that of Panel (b) of Figure 1 in the introduction. Firstly, comparing the case where there is no investor ($I_2 = 0$) to that where there is a single investor ($I_2 = 1$) exploiting the second characteristic, Part 3 of Proposition 4.5 shows that trading diversification reduces the price impact from exploiting the first characteristic, which leads to an increase in the capacity as well as in the equilibrium aggregate investment position and profits of the first characteristic. *Thus, investment in the second characteristic increases profits from the first characteristic because of trading diversification.*

Secondly, an increase in competition among investors exploiting the *second* characteristic, measured by an increase in I_2, further reduces the price impact from exploiting the first characteristic, and thus, increases the capacity as well as the equilibrium aggregate investment position and profits for the first characteristic. To understand this result, note that there is a negative externality among the investors exploiting the second characteristic because they do not internalize in their objective function the effect of their investment decisions on each other's price-impact costs. This externality worsens as I_2 increases and leads them to increase their aggregate investment position, which increases their price-impact costs and reduces their profits. However, the increase in the aggregate investment position of investors exploiting the second characteristic *reduces* the price impact experienced by investors exploiting the first characteristic because of trading diversification, which leads to an increase in their aggregate profits. *Thus, competition among investors exploiting the second characteristic further increases profits from the first characteristic because of the positive externality between the two groups of investors.*

4.4 Centralized investing in characteristics

Finally, we consider a centralized setting in which a *single* investor exploits both characteristics. The following proposition shows that centralization leads to an increase in the total profits from exploiting both characteristics compared to the case where a separate investor exploits each of the characteristics. The main takeaway from this result is that financial institutions have an incentive to *centralize* the utilization of multiple characteristics because of trading diversification. We use the subscript "c" to denote optimal investment positions and profits in the centralized setting.

Proposition 4.6 *Let Assumption 4.1 hold and consider a centralized setting in which a single investor exploits both characteristics and a decentralized setting in which a separate investor exploits each characteristic. Then:*

1. *The equilibrium investment position in the first characteristic in the centralized setting, θ_{1c}, is larger than in a decentralized setting; that is, $\theta_{1c} > \theta_{1d} > 0$.*

2. *The equilibrium investment position in the second characteristic in the centralized setting, θ_{2c}, is negative and smaller than in the decentralized setting; that is, $\theta_{2c} < \theta_{2d} < 0$.*

3. *The profits from trading the second characteristic π_{2c} are zero in the centralized setting and smaller than those in the decentralized setting; that is, $0 = \pi_{2c} < \pi_{2d}$.*

4. *The equilibrium total profits π_c and the equilibrium profits from the first characteristic in the centralized setting π_{1c} are larger than those in the decentralized setting; that is, $\pi_c > \pi_d$ and $\pi_{1c} > \pi_{1d}$.*

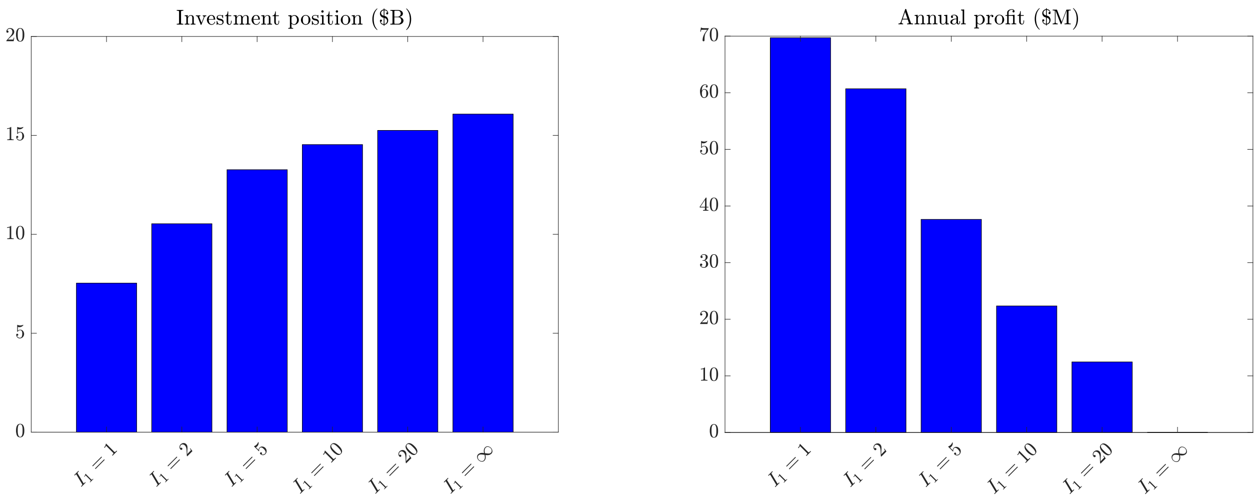
To understand the intuition underlying Proposition 4.6, note that centralizing the trading of two characteristics allows the single investor to internalize the positive externality present in the decentralized setting between the investor exploiting the first characteristic and the investor exploiting the second. In particular, the single investor in the centralized setting internalizes the reduction in the price-impact cost of exploiting the first characteristic that is achieved by increasing the investment position in the second characteristic. Consequently, the single investor increases the investment position in the second characteristic to the point where profits from the second characteristic are zero because this leads to a reduction in the price-impact costs from exploiting the first characteristic. This reduction then allows the single investor to increase investment in the first characteristic and maximize total profits.

4.5 Empirical calibration

To estimate the impact of competition across characteristics on the equilibrium, we calibrate the game-theoretic model using the historical stock-return and characteristic data that we downloaded for the analysis in Section 3. For our theoretical analysis in Sections 4.1–4.4, we assumed the three conditions in Assumption 4.1. For the empirical calibration in this

Figure 5: Competition in a single characteristic

This figure illustrates the effect of competition on aggregate investment positions and profits when there are only investors exploiting the first characteristic ($I_1 \geq 1$ and $I_2 = 0$). The figure depicts the aggregate investment position and profits for the cases with $I_1 = 1, 2, 5, 10, 20, \infty$ investors. We consider the price-impact cost model of Frazzini et al. (2018) and use "investment (asset growth)" as the characteristic.



section, we relax these three conditions. As in Section 3.2, we estimate price-impact costs using the model of Frazzini et al. (2018), which contains both linear ($\alpha = 1$) and square root ($\alpha = 0.5$) price-impact terms. For this price-impact cost model, there are no closed-form expressions for the equilibrium quantities, so we compute these numerically.[17] We use "investment (asset growth)" as the first characteristic and "gross profitability" as the second characteristic; Section IA.3 of the Internet Appendix shows that our findings are robust to considering other pairs of characteristics, such as: (i) "book to market" and "gross profitability," (ii) "momentum" and "gross profitability," and (iii) "momentum" and "book to market."

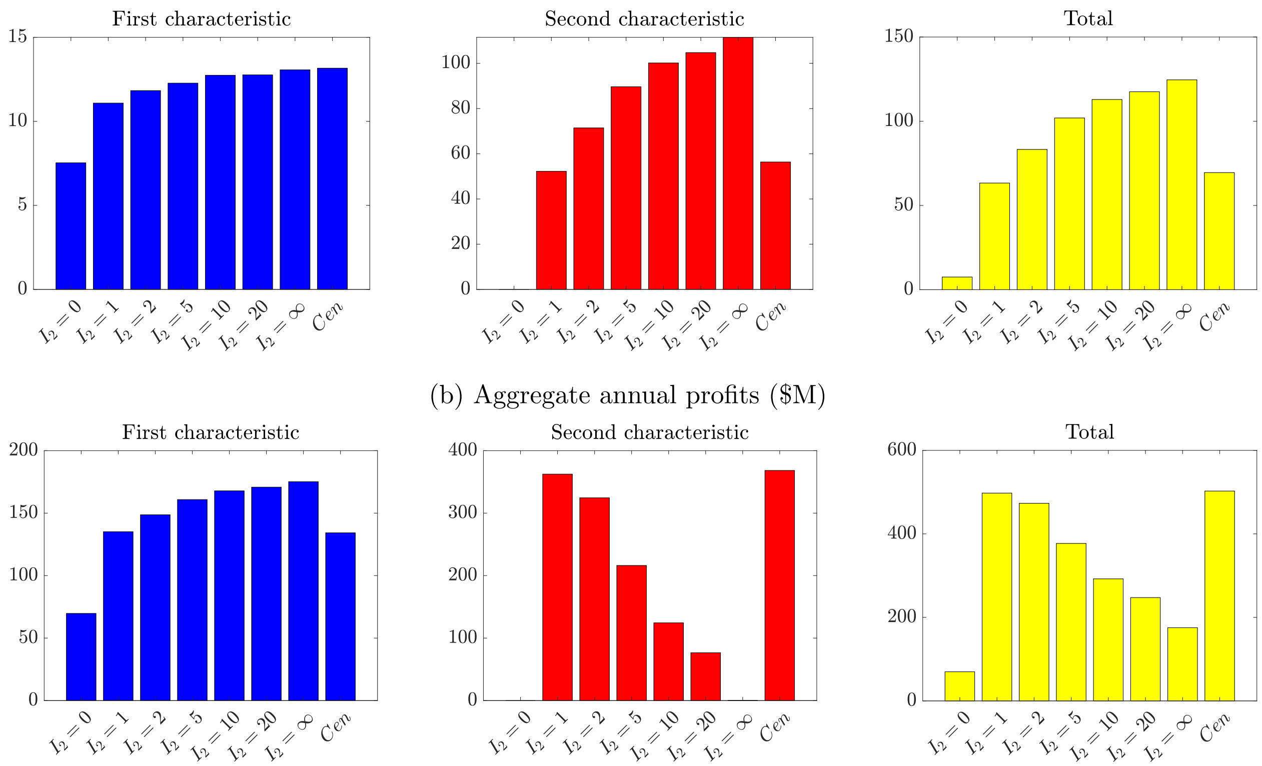
Figure 5 illustrates the effect of competition on aggregate investment positions and profits when there are only investors exploiting the first characteristic ($I_1 \geq 1$ and $I_2 = 0$). The figure depicts the aggregate investment position and profits for the first characteristic

[17] In particular, we use a best-response iterative procedure to find the equilibrium. As a starting point, we set the investment position of each investor equal to the optimal position in the centralized setting divided by the number of investors. Then, at each iteration of the procedure we first compute the best response of an investor exploiting the first characteristic who assumes the rest of the investors keep their investment positions equal to those in the previous iteration. We then compute the best response of an investor exploiting the second characteristic who assumes the rest of the investors exploiting the second characteristic keep their positions equal to those in the previous iteration and the investors exploiting the first characteristic keep their positions equal to their previously calculated best response. We stop this iterative procedure when the investment position of every investor in two consecutive iterations is sufficiently close. We use the equilibrium conditions to confirm that this procedure converges to a Nash equilibrium for all of our numerical experiments.

Figure 6: Competition in a second characteristic

This figure depicts the investment positions and profits for the two characteristics for the decentralized setting where there is a single investor exploiting the first characteristic $I_1 = 1$ and $I_2 = 0, 1, 2, 5, 10, 20, \infty$ investors competing to exploit the second, and also for the centralized setting (Cen). Panel (a) depicts the aggregate investment position in each of the two characteristics and the total investment position across both characteristics in billions of dollars. Panel (b) depicts the annual profits obtained from each characteristic and the total profits from both characteristics in millions of dollars. We consider the price-impact cost model of Frazzini et al. (2018) and use "investment (asset growth)" as the first characteristic and "gross profitability" as the second.

(a) Aggregate investment positions ($B)



for the cases with $I_1 = 1, 2, 5, 10, 20, \infty$ investors. We observe that increasing the number of investors competing to exploit the "investment (asset growth)" characteristic from $I_1 = 1$ to $I_1 = 20$ *doubles* their aggregate investment position, from \$7.5 billion to \$15 billion and, because of crowding, greatly reduces the aggregate expected annual profit, from almost \$70 million to just above \$10 million. In the limit, as the investors become perfectly competitive ($I_1 = \infty$), their aggregate investment position is 113% greater than that for the case with a single investor and their aggregate profits vanish.

Figure 6 depicts the investment positions and profits for the case with two characteristics for the decentralized setting where there is a single investor exploiting the first characteristic $I_1 = 1$ and $I_2 = 0, 1, 2, 5, 10, 20, \infty$ investors competing to exploit the second, and also for the centralized setting (Cen). Panel (a) depicts the aggregate investment position in each of the two characteristics and the total investment position across both characteristics. Panel (b) depicts the profits obtained from each characteristic and the total profits from both characteristics.

Comparing the case where there are no investors ($I_2 = 0$) to the case where there is a single investor exploiting the second characteristic ($I_2 = 1$), we observe from Figure 6 that the presence of a single investor exploiting the second characteristic leads to an increase of around 50% in the aggregate investment position in the first characteristic, from \$7.5 billion to around \$11.25 billion, and an increase in profits from the first characteristic of around 95%, from almost \$70 million to around \$136.5 million. Moreover, when the number of investors exploiting the second characteristic increases from $I_2 = 1$ to $I_2 = 20$, their aggregate annual profits are reduced by almost 80%, from around \$360 million to around \$75 million, and their aggregate investment position more than doubles, from around \$50 billion to around \$110 billion. This additional investment in the second characteristic generates trading diversification benefits for the investor exploiting the first characteristic "investment (asset growth)," who, in response, increases her aggregate investment by 15% and her aggregate profits by 26%. Overall, comparing the case without investors exploiting the second characteristic ($I_2 = 0$) to the case with twenty investors ($I_2 = 20$), competition to exploit the second characteristic leads to a 70% increase in aggregate investment position and a 145% increase in aggregate profits from the first characteristic.

Finally, the single investor in the centralized setting maximizes the *total* profits across the two characteristics by taking an even greater investment position in the first characteristic, but a smaller position in the second characteristic, compared to the decentralized setting with $I_1 = 1$ and $I_2 = 20$. This is because by reducing the investment position in the second characteristic, the single investor substantially increases the profits from the second characteristic because of the substantial reduction in price-impact costs at the expense of only a modest reduction in the profits from the first characteristic, thus generating substantially higher total profits.[18]

[18]Note that this result differs from the theoretical result for the centralized setting in Section 4.5, where we found that the single investor increased her position on the second characteristic until price-impact costs

Summarizing, the empirical calibration of our game-theoretic model shows that, because of the trading-diversification mechanism, competition across different characteristics has a substantial effect on the equilibrium investment positions and profits of financial institutions exploiting factor-investing strategies.

5 Conclusion

The explosion in the *number* of fund managers investing in factors has raised concerns about the effect of competition on the profitability of factor-investing strategies. The analysis in our manuscript suggests that the answer to the question posed in the title is that, through the *trading-diversification* mechanism, competition across different characteristics can increase profits in factor investing. Specifically, our game-theoretic model shows that, although competition among investors exploiting a *particular* characteristic does erode their profits, competition among investors exploiting *other* characteristics increases the profits of investors exploiting the first characteristic because of the positive externality generated by trading diversification. Our empirical analysis shows that competition across investors exploiting different characteristics has a substantial effect on their capacity and on the equilibrium investment positions and profits of financial institutions.

completely eroded profits from the second characteristic. The reason for this difference is that for the empirical analysis we have relaxed the assumption that the mean return of the second characteristic is zero.

A Trading-diversification example

In this appendix, we use an example to provide intuition for the difference in the theoretical results for the cases with subquadratic price-impact cost function ($\alpha = 0.5$) and quadratic cost function ($\alpha = 1$) in Section 3.1. In particular, we show that the theoretical findings are driven by the fact that the subquadratic price-impact cost function ($\alpha = 0.5$) assigns a *lower cost to large trades* than the quadratic function ($\alpha = 1$).

Consider a simple example with two characteristics whose rebalancing trades in the nth stock, $\tilde{x}_{1tn}$ and $\tilde{x}_{2tn}$, are independently and identically distributed with equal probability to take a value of -1 or $+1$. Moreover, let the price-impact parameter for the nth stock be constant and equal to one, $\lambda_{tn} = 1$.[19] Then, for the case with quadratic price-impact costs, the expected cost of rebalancing each of the characteristics in isolation is equal to one:

$$E\left[|\tilde{x}_{ktn}|^2\right] = \frac{1}{2}|-1|^2 + \frac{1}{2}|+1|^2 = 1 \quad \text{for} \quad k = 1, 2.$$

To calculate the price-impact cost of trading the two characteristics in combination, we consider four equally likely outcomes depending on the values of $\tilde{x}_{1tn}$ and $\tilde{x}_{2tn}$:

$$(\tilde{x}_{1tn}, \tilde{x}_{2tn}) = \begin{cases} (-1, -1), \\ (-1, +1), \\ (+1, -1), \\ (+1, +1). \end{cases}$$

Thus, the expected cost of trading the two characteristics in combination is equal to two:

$$\begin{aligned} E\left[|\tilde{x}_{1tn} + \tilde{x}_{2tn}|^2\right] &= \frac{1}{4} \times |-1-1|^2 + \frac{1}{4} \times |-1+1|^2 + \frac{1}{4} \times |+1-1|^2 + \frac{1}{4} \times |+1+1|^2 \\ &= \frac{1}{4} \times |-2|^2 + \frac{1}{4} \times |0|^2 + \frac{1}{4} \times |0|^2 + \frac{1}{4} \times |+2|^2 = 2. \end{aligned}$$

Thus, from (10) we have that

$$\text{price-impact diversification ratio} = \frac{E\left[|\tilde{x}_{1tn} + \tilde{x}_{2tn}|^2\right]}{E\left[|\tilde{x}_{1tn}|^2\right] + E\left[|\tilde{x}_{2tn}|^2\right]} = \frac{2}{1+1} = 1.$$

[19]We assume that the absolute value of the rebalancing trades and the price-impact parameter are equal to one without loss of generality as the price-impact diversification ratio is invariant to multiplying the rebalancing trades of all characteristics or the price-impact parameter by a constant because the absolute-value and power functions are homogeneous.

The price-impact diversification ratio is 1 for the case with quadratic costs because even though the price-impact cost is zero for the two outcomes where the trades of the two characteristics net out, $(-1,+1)$ and $(+1,-1)$, the *quadratic* price-impact costs of trading the characteristics in combination are large (equal to four) for the two outcomes where the rebalancing trades of the two characteristics are in the same direction, $(-1,-1)$ and $(+1,+1)$. In other words, the increase in price-impact cost from trading characteristics in combination for the two outcomes where the trades of the two characteristics are in the same direction exactly compensates for the reduction for the two outcomes where they net out. This is because the quadratic price-impact costs are disproportionately high for large trades.

For the square-root price-impact function, that is, subquadratic price-impact costs, the expected cost of rebalancing each of the characteristics in isolation is also equal to one:

$$E\left[|\tilde{x}_{ktn}|^{1.5}\right] = \frac{1}{2} \times |-1|^{1.5} + \frac{1}{2} \times |+1|^{1.5} = 1 \quad \text{for} \quad k = 1, 2.$$

In contrast, the expected cost of trading the two characteristics in combination is only $\sqrt{2}$:

$$\begin{aligned} E\left[|\tilde{x}_{1tn} + \tilde{x}_{2tn}|^{1.5}\right] &= \frac{1}{4} \times |-1-1|^{1.5} + \frac{1}{4} \times |-1+1|^{1.5} + \frac{1}{4} \times |+1-1|^{1.5} + \frac{1}{4} \times |+1+1|^{1.5} \\ &= \frac{1}{4} \times |-2|^{1.5} + \frac{1}{4} \times |0|^{1.5} + \frac{1}{4} \times |0|^{1.5} + \frac{1}{4} \times |+2|^{1.5} \\ &= \frac{2 \times 2^{1.5}}{4} = \sqrt{2}. \end{aligned}$$

Thus, we have that for the square-root price-impact function the

$$\text{price-impact diversification ratio} = \frac{E\left[|\tilde{x}_{1tn} + \tilde{x}_{2tn}|^{1.5}\right]}{E\left[|\tilde{x}_{1tn}|^{1.5}\right] + E\left[|\tilde{x}_{2tn}|^{1.5}\right]} = \frac{\sqrt{2}}{1+1} = \frac{1}{\sqrt{2}} < 1.$$

The price-impact diversification ratio is smaller than one for the case with square-root price impact because, in addition to having zero price-impact cost for the two outcomes where the rebalancing trades of the two characteristics cancel out, $(-1,+1)$ and $(+1,-1)$, the *subquadratic* price-impact costs of trading the characteristics in combination are only $2^{1.5}$ for the two outcomes where the rebalancing trades of the two characteristics are in the same direction, $(-1,-1)$ and $(+1,+1)$, compared to 2^2 for the case with quadratic costs.

B Game-theoretic model of strategic competition

In this appendix, we characterize the equilibrium of the game-theoretic model proposed in Section 4 for the more general case where Assumption 4.1(3) does not hold; that is, for the case where the mean return of the second characteristic is nonzero. We also provide a more detailed analysis of the solution to the game-theoretic model.

The portfolio of the ith investor exploiting the kth characteristic at time t is

$$w_{kit}(\theta_{ki}) = x_{kt}\theta_{ki},$$

where $\theta_{ki} \in \mathbb{R}$ is the investment position of the ith investor exploiting the kth characteristic. The portfolio return of the ith investor exploiting the kth characteristic at time $t+1$ is

$$r_{ki,t+1} = \theta_{ki} x_{kt}^{\top} r_{t+1}. \tag{B1}$$

The following lemma shows that the price-impact cost of investors is a quadratic function of their investment positions.

Lemma B.1 *Under Assumption 4.1(1), the price-impact cost of the ith investor exploiting the first and second characteristics at time t can be written as*

$$PIC_{1it} = \theta_{1i}\lambda_{1t}(\theta_{1i} + \theta_{1,-i}) + \theta_{1i}\lambda_{12t}\sum_{j=1}^{I_2}\theta_{2j} \quad \text{and} \tag{B2}$$

$$PIC_{2it} = \theta_{2i}\lambda_{2t}(\theta_{2i} + \theta_{2,-i}) + \theta_{2i}\lambda_{12t}\sum_{j=1}^{I_1}\theta_{1j}, \quad \text{respectively,} \tag{B3}$$

where $\theta_{k,-i} = \sum_{j\neq i}\theta_{kj}$ is the aggregate investment position of investors in the kth characteristic other than the ith investor and $\lambda_{kt} = \tilde{x}_{kt}^{\top}\Lambda_t\tilde{x}_{kt}$ and $\lambda_{12t} = \tilde{x}_{1t}^{\top}\Lambda_t\tilde{x}_{2t}$ are the price-impact parameters for the kth characteristic and the interaction between the two characteristics at time t, respectively, where Λ_t is the price-impact matrix at time t defined in (3) *and $\tilde{x}_{kt}$ is the rebalancing-trade vector for the kth characteristic at time t defined in* (7).

Lemma B.1 shows that, for the case with linear price-impact function, the trading costs for the two investors can be conveniently decomposed into three distinct terms. The terms associated with λ_{1t} and λ_{2t} measure the price-impact cost associated with exploiting in isolation the first and second characteristics, respectively. The parameter λ_{12t} measures the *interaction* between the rebalancing trades for the two characteristics. For $\lambda_{12t} = 0$, the price-impact costs of exploiting the two characteristics are independent, for $\lambda_{12t} < 0$ (> 0) there is a positive (negative) externality between the two groups of investors because trading in one characteristic decreases (increases) the price-impact cost of trading the other.

B.1 Decentralized setting

In the decentralized setting, we consider a game where the two groups of investors make decisions *simultaneously*; however, in unreported results we observe that our findings are robust to considering the case where investors in one of the groups act as Stackelberg leaders.

The ith investor in the kth characteristic chooses her investment position θ_{ki} to maximize the unconditional expectation of the difference between her portfolio return and price-impact cost:

$$\max_{\theta_{ki}} \quad E[r_{ki,t+1} - \text{PIC}_{kit}]. \tag{B4}$$

Then, the following proposition generalizes Proposition 4.2 for the case where the mean return of the second characteristic may be different from zero.

Proposition B.1 *Let Assumptions 4.1(1) and 4.1(2) hold. Then, the decision problems of the ith investors in the first and second characteristics can be written as*

$$\max_{\theta_{1i}} \quad \underbrace{\theta_{1i}\mu_1}_{\textit{mean return}} \underbrace{- \theta_{1i}\lambda_1\theta_{1i} - \overbrace{\theta_{1i}\lambda_1 \textstyle\sum_{j\neq i}\theta_{1j}}^{\textit{negative externality}} - \overbrace{\theta_{1i}\lambda_{12}\textstyle\sum_{j=1}^{I_2}\theta_{2j}}^{\textit{positive externality}}}_{\textit{price-impact cost}}, \quad \textit{and} \tag{B5}$$

$$\max_{\theta_{2i}} \quad \underbrace{\theta_{2i}\mu_2}_{\textit{mean return}} \underbrace{- \theta_{2i}\lambda_2\theta_{2i} - \overbrace{\theta_{2i}\lambda_2 \textstyle\sum_{j\neq i}\theta_{2j}}^{\textit{negative externality}} - \overbrace{\theta_{2i}\lambda_{12}\textstyle\sum_{j=1}^{I_1}\theta_{1j}}^{\textit{positive externality}}}_{\textit{price-impact cost}}, \quad \textit{respectively}, \tag{B6}$$

where $\lambda_k = E[\lambda_{kt}]$ is the price-impact parameter for the kth characteristic, $\lambda_{12} = E[\lambda_{12t}]$ is the price-impact parameter for the interaction between the two characteristics, and $\mu_k = E[x_{kt}^\top r_{t+1}]$ is the average return of the kth characteristic portfolio.

Note that although μ_1 and μ_2 are exogenous in our model, the average characteristic return *net of price-impact costs*, $\bar{\mu}_1$ and $\bar{\mu}_2$, are determined endogenously as a function of the investment positions. For instance, for the first characteristic we have $\bar{\mu}_1 = \mu_1 - \lambda_1(\theta_{1i} + \theta_{1,-i}) - \lambda_{12}\sum_{j=1}^{I_2}\theta_{2j}$.

B.2 Centralized setting

To understand the impact of competition between the two groups of investors, we also consider a centralized setting in which a single investor exploits both characteristics. For the case with linear price impact, the decision problem in the centralized setting is:

$$\max_{\theta_{1c},\theta_{2c}} \quad \theta_{1c}\mu_1 + \theta_{2c}\mu_2 - \theta_{1c}\lambda_1\theta_{1c} - 2\theta_{1c}\lambda_{12}\theta_{2c} - \theta_{2c}\lambda_2\theta_{2c}, \tag{B7}$$

where the subscript "c" denotes the optimal quantities for the centralized market. Note that because the objective function in the centralized setting is to maximize total profits, the total profit in the centralized setting is an upper bound for that in the decentralized setting.

B.3 Equilibrium

We now characterize the unique equilibrium in closed form for both the decentralized and centralized settings.

Under Assumptions 4.1(1) and 4.1(2), Lemma B.2 below shows using the triangular inequality that the absolute value of the price-impact parameter for the interaction between the two characteristics is bounded above.

Lemma B.2 *Let Assumptions 4.1(1) and 4.1(2) hold. Then,* $\lambda_1, \lambda_2 > 0$ *and the absolute value of the price-impact parameter for the interaction between the two characteristics is bounded above by* $\bar{\lambda}_{12} \equiv \sqrt{\lambda_1\lambda_2}$*; that is,* $|\lambda_{12}| < \bar{\lambda}_{12}$*.*

The following proposition generalizes Proposition 4.2 for the case where the mean return of the second characteristic is nonzero.

Proposition B.2 *Let Assumptions 4.1(1) and 4.1(2) hold. Then, in the decentralized setting we have that:*

1. *There exists a unique Nash equilibrium.*

2. *The equilibrium is symmetric with respect to the* I_1 *investors exploiting the first characteristic and with respect to the* I_2 *investors exploiting the second.*

3. *The investment positions of the ith investor exploiting the first characteristic and the ith investor exploiting the second characteristic are*

$$\theta_{1id} = \frac{(I_2+1)\,\lambda_2\mu_1 - I_2\lambda_{12}\mu_2}{(I_1+1)(I_2+1)\lambda_1\lambda_2 - I_1I_2\lambda_{12}^2} \quad \text{and} \tag{B8}$$

$$\theta_{2id} = \frac{(I_1+1)\lambda_1\mu_2 - I_1\lambda_{12}\mu_1}{(I_1+1)(I_2+1)\lambda_1\lambda_2 - I_1I_2\lambda_{12}^2}, \quad \text{respectively.} \tag{B9}$$

4. *The profit of the ith investor exploiting the kth characteristic is*

$$\pi_{kid} = \lambda_k\theta_{kid}^2. \tag{B10}$$

5. *The price-impact cost of the ith investor exploiting the kth characteristic is*

$$\zeta_{kid} = \mu_k\theta_{kid} - \pi_{kid} = (\mu_k - \lambda_k\theta_{kid})\theta_{kid}. \tag{B11}$$

6. *The price impact experienced by the ith investor exploiting the kth characteristic is*

$$\xi_{kid} = \frac{\zeta_{kid}}{\theta_{kid}} = \mu_k - \lambda_k\theta_{kid}. \tag{B12}$$

The following proposition gives the optimal investments and profit in the centralized setting, denoted by the subscript "c".

Proposition B.3 *Let Assumptions 4.1(1) and 4.1(2) hold. Then, in the centralized setting we have that:*

1. *There exists a unique maximizer to the centralized decision problem.*

2. *The optimal investment positions are*

$$\theta_{1c} = \frac{\lambda_2\mu_1 - \lambda_{12}\mu_2}{2(\lambda_1\lambda_2 - \lambda_{12}^2)}, \tag{B13}$$

$$\theta_{2c} = \frac{\lambda_1\mu_2 - \lambda_{12}\mu_1}{2(\lambda_1\lambda_2 - \lambda_{12}^2)}. \tag{B14}$$

3. *The price-impact costs from trading the first and second characteristics, ζ_{1c} and ζ_{2c}, coincide with the profits from the first and second characteristics, π_{1c} and π_{2c}, and are*

$$\zeta_{1c} = \pi_{1c} = \frac{\frac{1}{2}\lambda_2\mu_1^2 - \frac{1}{2}\lambda_{12}\mu_1\mu_2}{2(\lambda_1\lambda_2 - \lambda_{12}^2)}, \tag{B15}$$

$$\zeta_{2c} = \pi_{2c} = \frac{\frac{1}{2}\lambda_1\mu_2^2 - \frac{1}{2}\lambda_{12}\mu_1\mu_2}{2(\lambda_1\lambda_2 - \lambda_{12}^2)}. \tag{B16}$$

4. The price impacts from exploiting the first and second characteristics, ζ_{1c} and ζ_{2c}, coincide with the returns net of price impact from exploiting the first and second characteristics, $\bar{\mu}_{1c}$ and $\bar{\mu}_{2c}$, and are equal to half the gross return from the first and second characteristics, μ_1 and μ_2:

$$\xi_{1c} = \bar{\mu}_{1c} = \frac{\mu_1}{2}, \tag{B17}$$

$$\xi_{2c} = \bar{\mu}_{2c} = \frac{\mu_2}{2}. \tag{B18}$$

C Empirical tests using mutual-fund data

We now provide empirical support for the two key predictions of our game-theoretic model using data on mutual-fund holdings, stock returns, and firm characteristics. The first key prediction of our model is that competition among investors exploiting the *same* characteristic erodes their profits because of crowding. In particular, Part 3 of Proposition 4.3 predicts that the price-impact cost of investors exploiting a characteristic increases with the number of investors, which measures the intensity of competition. As a consequence of this increase in price-impact costs, Part 2 of Proposition 4.3 predicts that aggregate profits decrease with the number of investors.

The second key prediction of our model is that competition among investors exploiting *other* characteristics alleviates crowding because of trading diversification. In particular, Part 3 of Proposition 4.5 predicts that the price impact experienced by investors exploiting a characteristic decreases with an increase in the number of investors exploiting a *second* characteristic, that is, with the intensity of competition to exploit the second characteristic. Consequently, Part 2 of Proposition 4.5 predicts that the aggregate profit from exploiting a characteristic *increases* with the number of investors exploiting a second characteristic.

To provide support for these predictions, we run regressions of the returns of each characteristic on a novel measure of competition to buy the same characteristic as well as a second characteristic. We highlight two findings. First, a unit increase in competition to buy *just* the first characteristic leads to a contemporaneous return of the first characteristic of around 40%, with the cumulative return reverting fully over time. This return reversal shows that investors exploiting a characteristic that experiences high buy competition suffer large price-impact costs, and thus, provides support for the first prediction of our model that crowding erodes profits. Second, the contemporaneous return response of the first characteristic to a simultaneous unit increase in competition to buy *both* the first characteristic *and* the second characteristic is about 17% smaller than that to a unit increase in competition to buy *only* the first characteristic. This evidence supports the second prediction of our model that competition among investors exploiting *other* characteristics alleviates crowding because of trading diversification.

Our empirical approach follows the literature that studies the impact of nondiscretionary mutual-fund trading on characteristic returns. Huang, Song, and Xiang (2021) construct a *characteristic-level* measure of nondiscretionary mutual-fund trading by aggregating the stock-level flow-induced trading (FIT) of Lou (2012) and show that mutual funds' flow-induced trades of factors do not carry information about fundamentals. Li (2021) exploits the characteristic-level FIT to show that non-discretionary trading exerts price pressure on the size and value characteristics. To do this, he uses panel regressions to show that a 1% increase in characteristic FIT is associated with a contemporaneous increase in characteristic returns of 2% that reverts fully over time.[20] Unlike Li (2021), our focus is on how *competition* to exploit different characteristics affects price-impact costs, and thus, instead of relying on measures of nondiscretionary trading, we define a novel measure of buying and selling competition at the characteristic level.

The remainder of this appendix is organized as follows. Section C.1 describes the data and our measure of competition to buy each characteristic, Section C.2 gives the results from regressing the returns of each characteristic on measures of competition to buy the same characteristic and a second characteristic, and Section C.3 reports various robustness checks.

C.1 Data and measures of competition

We download quarterly holdings of US equity mutual funds from Thomson Reuters. Like Doshi, Elkamhi, and Simutin (2015), we combine all share classes issued by each fund and drop funds with ten or fewer stock holdings or less than $15 million of assets under management. We merge the resulting mutual-fund holdings with the stock returns and characteristics that we downloaded for the analysis in Section 3.[21] Our sample spans the period from January 1980 to December 2018 and contains data for 15,238 stocks and 3,516 funds.[22]

We first define a measure of competition among mutual funds to buy and sell *individual stocks* and then we aggregate it to the characteristic level. In our theoretical analysis in Section 4, we measure competition using the number of investors exploiting each character-

[20]In addition, (Li, 2021, Section 4.2) runs a structural vector autoregression to capture the joint dynamics of characteristic FIT and returns and finds that "1% of buying pressure in a factor creates a price impact of 5.7%" and that characteristic FIT "explains around 30% of the size and value factor price fluctuations."

[21]As explained in Section 3, we form value-weighted long-short portfolios for each characteristic by going long on stocks with values of the characteristic above the 70th percentile and going short stocks with values of the characteristic below the 30th percentile. In practice, institutional investors use a variety of characteristic portfolios tailored to their specific goals, but we use a 70/30 value-weighted portfolio because it provides a good tradeoff between average gross return and price-impact costs.

[22]We thank Mikhail Simutin for sharing the code to replicate the results of Doshi et al. (2015).

istic, which is appropriate because the equilibrium to our game-theoretic model is symmetric with respect to all investors exploiting each characteristic, as shown in Proposition 4.2. To measure competition using mutual-fund data, we have to account for the asymmetry across mutual funds, and thus, we rely on the Herfindahl-Hirschman index (HHI) of concentration in stock purchases to construct a measure of competition. Note that for the case with a symmetric equilibrium, there is a one-to-one correspondence between HHI and number of firms.[23]

To define the measure for individual stocks, we first compute the number of shares of stock n bought by fund j in quarter t as $b_{jtn} = [h_{jtn} - h_{j,t-1,n}]^+$, where h_{jtn} is the number of shares of stock n held by fund j in quarter t, and $[x]^+ = \max(x, 0)$ is the positive part of x. Similarly, we compute the number of shares sold as $s_{jtn} = [h_{jtn} - h_{j,t-1,n}]^-$, where $[x]^- = \max(-x, 0)$ is the negative part of x. Then, we estimate the competition among funds to buy stock n in quarter t as

$$\text{BuyComp}_{tn} = \Bigg(1 - \underbrace{\frac{\sum_j b_{jtn}^2}{(\sum_j b_{jtn})^2}}_{\substack{\text{Purchase}\\\text{concentration}}}\Bigg) \times \underbrace{\frac{\sum_j b_{jtn}}{\sum_j (b_{jtn} + s_{jtn})}}_{\substack{\text{Fraction of}\\\text{shares purchased}}}. \tag{C1}$$

The first term on the right-hand side of Equation (C1) is one minus the Herfindahl-Hirschman index (HHI), which measures concentration of stock purchases across funds. For instance, if a single fund buys all shares purchased, then the HHI is equal to one and the first term is zero (low buy competition). If, on the other hand, ten funds purchase an equal number of shares, then the HHI is 0.1 and the first term takes a value of 0.9 (high buy competition). Importantly, the HHI also measures the intensity of competition for asymmetric equilibria. For instance, if one fund buys a large number of shares (say 10,000) and nine funds buy a small number of shares (say 100 each), then the HHI is close to one and the first term is small (low competition). The second term on the right-hand side of Equation (C1) scales this competition measure by the fraction of all transactions that correspond to purchases instead of sales. Thus, if there are ten funds buying a small number of shares (10 shares each) and two funds selling a much larger number of shares (10,000 each), then the competition to buy is small relative to the competition to sell. Similarly, we

[23]The HHI of concentration is a popular measure of competition among firms; see, for instance, Rhoades (1993). In a mutual-fund context, Feldman et al. (2020) use the HHI of assets under management to measure competition.

estimate the competition to sell stock n at quarter t as

$$\text{SellComp}_{tn} = \left(1 - \frac{\sum_j s_{jtn}^2}{(\sum_j s_{jtn})^2}\right) \times \frac{\sum_j s_{jtn}}{\sum_j (b_{jtn} + s_{jtn})}. \tag{C2}$$

We then aggregate the competition measures at the characteristic level. Competition to buy a characteristic means that there is competition to *buy* the stocks in its long leg and *sell* the stocks in its short leg. Thus, we define the kth characteristic buy competition measure as the value-weighted buy competition of the stocks in its long leg plus the value-weighted sell competition of the stocks in its short leg:

$$\text{BuyComp}_{kt} = \sum_{n=1}^{N} \left([x_{ktn}]^+ \times \text{BuyComp}_{tn} + [x_{ktn}]^- \times \text{SellComp}_{tn}\right),$$

where x_{ktn} is the weight of the kth characteristic portfolio in stock n in quarter t and BuyComp_{tn} and SellComp_{tn} are the buy and sell competition measures of stock n in quarter t, respectively. Finally, in Section C.3.4 we show that our analysis is robust to constructing competition measures based only on the first of the two terms in (C1) and (C2).

C.2 Regression results

We estimate a pooled regression that measures trading diversification across all *pairwise* combinations of the 18 characteristics listed in Table 1. In particular, we regress the quarterly returns of each characteristic on contemporaneous and past values of competition to buy the same characteristic and competition to buy a second characteristic:

$$\begin{aligned} r_{k_1,t} &= a_0 \text{BuyComp}_{k_1,t} + a_1 \text{BuyComp}_{k_1,t-1} + \ldots + a_{12} \text{BuyComp}_{k_1,t-12} \\ &\quad + b_0 \text{BuyComp}_{k_2,t} + b_1 \text{BuyComp}_{k_2,t-1} + \ldots + b_{12} \text{BuyComp}_{k_2,t-12} \\ &\quad + \text{Controls}_{k_1,t} + u_{k_1,t}, \end{aligned} \tag{C3}$$

where $r_{k_1,t}$ is the return of characteristic k_1 in quarter t, $\text{BuyComp}_{k_1,t}$ and $\text{BuyComp}_{k_2,t}$ are the measures of competition to buy characteristics k_1 and k_2 in quarter t, respectively, and the $\text{Controls}_{k_1,t}$ include past returns for characteristic k_1 up to 12 lags and characteristic fixed effects. The slope coefficients a_0 and b_0 measure the *contemporaneous* return response of characteristic k_1 to a unit increase in competition to buy characteristics k_1 and k_2, respectively, and a_i and b_i measure the return response after i quarters.

Table C.1: Regression slope coefficients

This table reports slope coefficients and t-statistics of the pooled regression model defined in Equation (C3) considering all pairwise combinations of the 18 characteristics listed in Table 1.

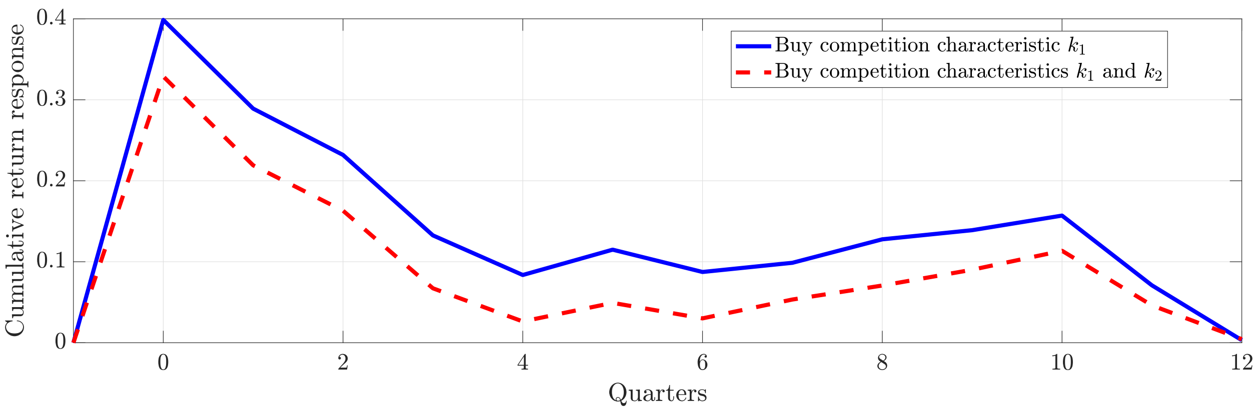
	Slope	t-stat		Slope	t-stat
a_0	0.399	55.684	b_0	−0.070	−10.738
a_1	−0.110	−14.314	b_1	−0.000	−0.018
a_2	−0.057	−6.677	b_2	0.001	0.168
a_3	−0.099	−11.216	b_3	0.003	0.487
a_4	−0.049	−6.223	b_4	0.008	1.164
a_5	0.031	4.152	b_5	−0.008	−1.239
a_6	−0.028	−3.753	b_6	0.008	1.212
a_7	0.011	1.632	b_7	0.012	1.791
a_8	0.029	3.657	b_8	−0.012	−1.820
a_9	0.011	1.713	b_9	0.008	1.261
a_{10}	0.018	2.795	b_{10}	0.005	0.821
a_{11}	−0.086	−12.911	b_{11}	0.019	3.107
a_{12}	−0.068	−11.843	b_{12}	0.026	5.172

Table C.1 reports the slope coefficients and t-statistics for the pooled regression model. We observe that the contemporaneous return response of characteristic k_1 to a unit increase in competition to buy characteristics k_1 and k_2 is statistically significant; that is, the t-statistics for a_0 and b_0 are significant.[24] To interpret the results from the panel regressions, see Figure C.1, in which the solid blue line depicts the cumulative return response of characteristic k_1 to a unit increase in competition to buy just that characteristic and the dashed red line to a simultaneous unit increase in competition to buy *both* that characteristic and a second characteristic k_2. Specifically, for each quarter q on the horizontal axis, the solid blue line depicts $\sum_{i=0}^{q} a_i$ and the dashed red line depicts $\sum_{i=0}^{q}(a_i + b_i)$.

We highlight two findings from Figure C.1. First, the solid blue line shows that characteristic k_1 experiences a 40% return contemporaneously with a unit increase in competition to buy just that characteristic, but its cumulative return reverts completely over time. This return reversal shows that characteristics that experience high buy competition suffer large

[24] In addition, the t-statistic for the slope corresponding to the first four lagged measures of competition to buy the first characteristic are also significant. Not surprisingly, the slope coefficients for the lagged measures of competition to buy the second characteristic are less significant than those for the first characteristic, because competition to buy the second characteristic affects the returns of the first characteristic only indirectly via trading diversification.

Figure C.1: Cumulative return response to buy competition in two characteristics
This figure depicts the cumulative return response of characteristic k_1 to a unit increase in competition to buy just that characteristic (solid blue line) and to a simultaneous unit increase in competition to buy both that characteristic and a different characteristic k_2 (dashed red line). The horizontal axis gives the time in quarters and the vertical axis depicts the cumulative return response. The pooled regression defined in Equation (C3) considers all pairwise combinations of the 18 characteristics listed in Table 1.



price-impact costs, and thus, provides support for the first prediction of our model that crowding erodes profits because of price-impact costs.

Second, comparing the dashed red line to the solid blue line in Figure C.1, we find that the return reversal experienced by characteristic k_1 after a simultaneous increase in competition to buy *both* characteristics k_1 and k_2 is significantly smaller than that after an increase in competition to buy *just* characteristic k_1. In particular, the *contemporaneous* return response of characteristic k_1 to a unit increase in competition to buy *both* characteristics k_1 and k_2 (given by $a_0 + b_0$) is 17.5% smaller than that associated with a unit increase in competition to buy *just* characteristic k_1 (given by a_0). Moreover, the difference between these two contemporaneous returns, the slope coefficient b_0, is highly statistically significant with a t-stat of -10.738 as shown in Table C.1. This result provides support for the second prediction of our model that investors exploiting a characteristic experience smaller price-impact costs when there are investors competing to exploit a different characteristic at the same time.

Several findings in the existing literature provide additional support for the predictions of our game-theoretic model. For instance, our first prediction is supported by Lou and Polk (2022) and Hoberg et al. (2020) in the context of the momentum characteristic. Lou and Polk (2022) propose a novel comomentum measure of arbitrage activity and find that when "comomentum is high, the returns on momentum stocks strongly revert, reflecting prior overreaction from crowded momentum trading which pushes prices away from fundamentals." Figure 3 in Lou and Polk (2022) shows that momentum stocks with high comomentum have smaller returns than those with low comomentum and that their returns revert strongly after just six months. Hoberg et al. (2020) show that momentum produces abnormal returns only when the momentum portfolio is constructed from stocks held by funds that do not face intense competition.

The second prediction of our game-theoretic model is also supported by the literature on fund herding (Wermers, 1999; Dasgupta, Prat, and Verardo, 2011), which shows that the magnitude or sign of the *aggregate* trade of institutional investors in a particular stock predicts the stock's short-term returns. Importantly, these papers rely on the aggregate *net* trade (purchases minus sales) to explain subsequent returns, and thus take into account trading diversification.

C.3 Robustness checks

This section checks the robustness of our empirical findings using mutual-fund data to considering two subsamples covering the first and second half of our dataset, controlling for flow-induced trading (FIT) in our regression analysis, constructing competition measures based on the nondiscretionary trades of mutual funds captured by FIT, constructing competition measures based only on purchase and sale concentration, and considering build-up (divergent) and resolve (converngent) characteristics separately. Our findings are summarized in Table C.2.

C.3.1 Subsample analysis

We first study the robustness of our empirical analysis based on mutual-fund data to considering two subsamples covering the first and second half of our overall sample. The second and third rows of Table C.2 show that the findings from regression (C3) are robust: the contemporaneous return response of characteristic k_1 to a simultaneous unit increase in competition to buy *both* characteristics k_1 and k_2 is smaller (for both subsamples) than that to a unit increase of competition to buy only characteristic k_1. Specifically, it is around 15% smaller for the first subsample and 9% smaller for the second subsample, with the difference being statistically significant for both subsamples (t-stat of b_0 greater than two).

Table C.2: Robustness checks for pooled regression (C3)

For each robustness check, each row of this table reports the results for the pooled regression (C3) of the return of characteristic k_1 on contemporaneous and past values of buy competition for characteristics k_1 and k_2. The first column lists the particular robustness check, the second column reports the contemporaneous return of characteristic k_1 in response to a unit increase in buy competition for characteristic k_1 (the slope coefficient a_0), the third column reports the contemporaneous return of characteristic k_1 in response to a simultaneous unit increase in buy competition for both characteristics k_1 and k_2 (the sum of the slope coefficients $a_0 + b_0$), the fourth column reports the percentage reduction in the contemporaneous return of characteristic k_1 from having a unit increase in competition for *both* characteristics k_1 and k_2 compared to having *only* a unit increase in buy competition for characteristic k_1 (the effect of trading diversification $-b_0/a_0$), and the fifth column reports the t-statistic for the slope coefficient b_0, which measures the effect of trading diversification between characteristics k_1 and k_2.

Robustness check	Contemp. return buy k_1, (%)	Contemp. return buy k_1 and k_2, (%)	Reduction (%)	t-stat
Base case	39.865	32.878	17.527	-10.738
First subsample	32.236	27.426	14.921	-7.046
Second subsample	35.050	31.741	9.441	-3.570
Controlling FIT	32.931	27.132	17.610	-9.370
Nondiscretionary comp.	32.547	28.802	11.506	-5.278
HHI comp.	15.256	9.966	34.675	-6.287
Build-up char.	44.550	37.211	16.474	-2.650
Resolve char.	56.524	46.758	17.278	-3.213

C.3.2 Controlling for flow-induced trading (FIT)

We now show that the findings from the analysis of mutual-fund holdings are robust to controlling for the characteristic-level FIT measure considered by Huang et al. (2021) and Li (2021). In particular, we re-estimate the pooled regression (C3) after controlling for contemporaneous and lagged values of characteristic FIT up to 12 quarters.

The fourth row of Table C.2 shows that the findings from regression (C3) are robust to controlling for characteristic FIT: the contemporaneous return response of characteristic k_1 to a simultaneous unit increase of competition to buy *both* characteristics k_1 and k_2 is 17.6% smaller than that to a unit increase in competition to buy *only* characteristic k_1 even after controlling for FIT, with the difference being statistically significant.

C.3.3 Nondiscretionary trading competition

The analysis in Section C.2 relies on measures of buy and sell competition derived from the *total* rebalancing trades of mutual funds, which capture competition in terms of the combination of both nondiscretionary and discretionary trades. We now study whether our results are robust to considering measures of competition obtained using *only* the nondiscretionary trades of mutual funds, captured by the flow-induced trading (FIT) of Lou (2012). In particular, we define measures of nondiscretionary buy competition as

$$\text{BuyComp}_{tn} = \left(1 - \frac{\sum_j \widetilde{b}_{jtn}^2}{(\sum_j \widetilde{b}_{jtn})^2}\right) \times \frac{\sum_j \widetilde{b}_{jtn}}{\sum_j (\widetilde{b}_{jtn} + \widetilde{s}_{jtn})}, \tag{C4}$$

where $\widetilde{b}_{jtn} = [h_{j,t-1,n} \times Flow_{jt}]^+$, $\widetilde{s}_{jtn} = [h_{j,t-1,n} \times Flow_{jt}]^-$, h_{jtn} is the number of shares of stock n held by fund j in quarter t, and $Flow_{jt}$ is the capital flow into fund j in quarter t, as defined by Lou (2012). The measure of nondiscretionary sell competition among mutual funds is defined similarly. We also aggregate the nondiscretionary BuyComp and SellComp measures to the characteristic level, as in Section C.1. We then repeat regression (C3), but using nondiscretionary competition measures instead of total competition measures.

The fifth row of Table C.2 shows that the findings from regression (C3) are robust to considering nondiscretionary competition measures: the contemporaneous return response of characteristic k_1 to a simultaneous unit increase in competition to buy *both* characteristics k_1 and k_2 is 11.5% smaller than that to a unit increase in competition to buy *only* characteristic k_1, with the difference being statistically significant.

C.3.4 Herfindahl-Hirschman-index (HHI) competition

The analysis in Section C.2 relies on measures of buy and sell competition given in Equations (C1) and (C2), respectively, which are the product of two terms. The first term is equal to one minus the HHI index, which captures the purchase or sale concentration, and the second term is the fraction of shares purchased or sold. In this section we study the robustness of our results to defining buy and sell competition measures based only on the first of these two terms; that is, based only on trade concentration:

$$\text{BuyComp}_{tn} = \Bigg(1 - \underbrace{\frac{\sum_j b_{jtn}^2}{(\sum_j b_{jtn})^2}}_{\substack{\text{Purchase}\\ \text{concentration}}}\Bigg) \quad \text{and} \quad \text{SellComp}_{tn} = \Bigg(1 - \underbrace{\frac{\sum_j s_{jtn}^2}{(\sum_j s_{jtn})^2}}_{\substack{\text{Sale}\\ \text{concentration}}}\Bigg).$$

The sixth row of Table C.2 shows that the findings from regression (C3) are robust to considering competition measures based only on trade concentration: the contemporaneous return response of characteristic k_1 to a simultaneous unit increase in competition to buy *both* characteristics k_1 and k_2 is about 34.7% smaller than that to a unit increase in competition to *only* buy characteristic k_1, with the difference being statistically significant.

C.3.5 Divergent versus convergent characteristics

van Binsbergen, Boons, Opp, and Tamoni (2021) classify characteristics as build-up (divergent) characteristics, whose returns exacerbate mispricing, and resolve (convergent) characteristics, whose returns alleviate mispricing. Although our focus is on how competition in the presence of trading diversification affects the profitability of factor-investing strategies, and thus, we are interested in characteristic net returns rather than characteristic mispricing, we now test the robustness of our results to considering build-up versus resolve characteristics. To do this, we split our characteristics into build-up and resolve groups based on the classification provided in Table 1 of van Binsbergen et al. (2021). Thus, our set of build-up characteristics contains {gma, mom12m, pchgm_pchsale, retvol, std_turn}, and our set of resolve characteristics contains {bm, bm_ia, mve, agr, beta}.

The last two rows of Table C.2 show that the findings from regression (C3) are robust to considering build-up and resolve characteristics separately. In particular, we find that the contemporaneous return response of characteristic k_1 to a simultaneous unit increase in competition to buy *both* characteristics k_1 and k_2 is about 16.5% and 17.3% smaller than that to a unit increase in competition to *only* buy characteristic k_1, for the cases with build-up and resolve characteristics, respectively, with the differences being statistically significant for both cases.

In unreported results, we also find that the overall shape of the return reversal over time for build-up and resolve characteristics is quite similar to that for the entire group of characteristics depicted in Figure C.1. This suggests that the characteristic return reversals that we document capture temporary price-impact costs.

D Proofs for all results

In this appendix we provide the proofs for all the results in the main body of the manuscript.

Proof of Proposition 3.1

For the case where the nth stock price-impact parameter is independently distributed from the rebalancing trades, the price-impact diversification ratio simplifies to

$$\text{price-impact diversification ratio} = \frac{E\left[\left|\sum_{k=1}^{K} \tilde{x}_{ktn}\right|^{1+\alpha}\right]}{\sum_{k=1}^{K} E\left[\left|\tilde{x}_{ktn}\right|^{1+\alpha}\right]}. \tag{D1}$$

Below, we characterize the expectation in the numerator and denominator on the right-hand side of Equation (D1) for $\alpha > -1$. Let

$$\tilde{x}_{tn}^{ew} = \sum_{k=1}^{K} \tilde{x}_{ktn}$$

be the trade in the nth stock required to rebalance an equally weighted portfolio of the K characteristics. Because $\tilde{x}_{ktn}$ for $k = 1, 2, \ldots, K$ are jointly distributed as a multivariate Normal distribution with zero mean and covariance matrix Ω, we have that $\tilde{x}_{tn}^{ew}$ is distributed as a Normal distribution with zero mean and variance $\sum_{k=1}^{K} \sigma_k^2 + \sum_{k=1}^{K} \sum_{l \neq k} \rho_{kl} \sigma_k \sigma_l$.

We need to characterize $E[|\tilde{x}_{tn}^{ew}|^{1+\alpha}]$. Because $\tilde{x}_{tn}^{ew}$ is distributed as a Normal distribution with zero mean, we have that $E[|\tilde{x}_{tn}^{ew}|^{1+\alpha}]$ is the central moment of order $1 + \alpha$ of a Normal random variable. Winkelbauer (2012) shows that for $\alpha > -1$

$$E[|\tilde{x}_{tn}^{ew}|^{1+\alpha}] = \frac{\Gamma(\frac{2+\alpha}{2})}{\pi} \times 2^{1+\alpha} \times \left(\sum_{k=1}^{K} \sigma_k^2 + \sum_{k=1}^{K} \sum_{l \neq k} \rho_{kl} \sigma_k \sigma_l\right)^{\frac{1+\alpha}{2}}, \tag{D2}$$

where $\Gamma(.)$ is the Gamma function; see Winkelbauer (2012, p. 1). Similarly, we have that

$$E[|\tilde{x}_{ktn}|^{1+\alpha}] = \frac{\Gamma(\frac{2+\alpha}{2})}{\pi} \times 2^{1+\alpha} \times \sigma_k^{1+\alpha}. \tag{D3}$$

Taking the ratio of (D2) to the summation of (D3) for $k = 1, 2, \ldots, K$, we obtain

$$\text{price-impact diversification ratio} = \frac{\left(\sum_{k=1}^{K} \sigma_k^2 + \sum_{k=1}^{K} \sum_{l \neq k} \rho_{kl} \sigma_k \sigma_l\right)^{\frac{1+\alpha}{2}}}{\sum_{k=1}^{K} \sigma_k^{1+\alpha}}. \tag{D4}$$

For the case with $\sigma_k^2 = \sigma^2$ for all k and $\rho_{kl} = \rho$ for all $k \neq l$, we have that

$$\text{price-impact diversification ratio} = \frac{\left[K(1 + (K-1)\rho)\right]^{\frac{1+\alpha}{2}}}{K}, \tag{D5}$$

where the term $K(1 + (K-1)\rho)$ is strictly positive because Ω is positive definite. Finally, the value of $\bar{\rho}$, which is defined in (12), follows using straightforward algebra.

Proof of Proposition 4.1

Proposition B.1 in Appendix B proves the more general result when Assumption 4.1(3) does not hold; that is, when the mean return of the second characteristic is nonzero. Therefore, Proposition 4.1 follows by simply setting $\mu_2 = 0$ in Equations (B5) and (B6).

Proof of Proposition 4.2

Proposition B.2 in Appendix B proves the more general result when Assumption 4.1(3) does not hold; that is, when the mean return of the second characteristic is nonzero. Therefore, Proposition 4.2 follows by simply setting $\mu_2 = 0$ in Equations (B8) and (B9).

Proof of Proposition 4.3

Note that the decision problems of the ith investor in the first characteristic in the *absence* and *presence* of investors in the second characteristic are identical for the case with $\lambda_{12} = 0$. Therefore, the equilibrium investment position, profit, and price-impact cost of the ith investor in the first characteristic in the absence of investors in the second characteristic are obtained by setting $\lambda_{12} = 0$ in Equations (16)–(19) of Proposition 4.2.

The monotonicity results follow from Equations (21) and (22) by noting that $I_1/(I_1 + 1)$ is increasing and $I_1/(I_1 + 1)^2$ is decreasing in I_1 for all $I_1 \geq 1$, and that the limits when I_1 goes to infinity of $I_1/(I_1 + 1)$ and $I_1/(I_1 + 1)^2$ are one and zero, respectively.

Proof of Proposition 4.4

To obtain the capacity of the first characteristic, we first determine the best response of the investors in the second characteristic to a given aggregate investment position in the first characteristic θ_{1d}. From Equation (15), the decision problem of the ith investor in the second characteristic is

$$\min_{\theta_{2i}} \quad \theta_{2i}\lambda_2(\theta_{2i} + \theta_{2,-i}) + \theta_{2i}\lambda_{12}\theta_{1d},$$

where $\theta_{2,-i} = \sum_{j \neq i} \theta_{2,j}$ is the aggregate investment position of investors in the second characteristic other than the ith investor. Thus, the first-order optimality condition for the ith investor in the second characteristic is

$$2\lambda_2\theta_{2i} + \lambda_2\theta_{2,-i} = -\lambda_{12}\theta_{1d}.$$

It follows from the proofs of Propositions 4.2 and B.2 that the equilibrium among investors in the second characteristic is symmetric, and thus we can rewrite the first-order optimality conditions as

$$(I_2 + 1)\lambda_2\theta_{2i} = -\lambda_{12}\theta_{1d},$$

and therefore the aggregate best response of the investors in the second characteristic is

$$\theta_{2d} = -\frac{I_2}{I_2 + 1}\frac{\lambda_{12}}{\lambda_2}\theta_{1d}. \tag{D6}$$

The capacity of the first characteristic is the aggregate investment position in the first characteristic for which its aggregate profits are zero, which must satisfy the following equation:

$$\theta_{1d}\lambda_1\theta_{1d} + \theta_{1d}\lambda_{12}\theta_{2d} - \theta_{1d}\mu_1 = 0.$$

We can simplify this equation by removing the trivial root $\theta_{1d} = 0$ to obtain

$$\lambda_1\theta_{1d} + \lambda_{12}\theta_{2d} - \mu_1 = 0.$$

Plugging (D6) into this equation we obtain that the capacity of the first characteristic is

$$\theta_{1d} = \frac{\mu_1}{\lambda_1 - \frac{I_2}{I_2+1}\frac{\lambda_{12}^2}{\lambda_2}}.$$

Proof of Proposition 4.5

Part 1. The partial derivative of the aggregate investment position in the first characteristic with respect to I_2 is

$$\frac{\partial\theta_{1d}}{\partial I_2} = \frac{\partial(I_1\theta_{1id})}{\partial I_2} = I_1\frac{\partial\theta_{1id}}{\partial I_2}$$

$$= I_1\frac{\lambda_2\mu_1\Big((I_1+1)(I_2+1)\lambda_1\lambda_2 - I_1I_2\lambda_{12}^2\Big) - (I_2+1)\lambda_2\mu_1\Big((I_1+1)\lambda_1\lambda_2 - I_1\lambda_{12}^2\Big)}{\Big((I_1+1)(I_2+1)\lambda_1\lambda_2 - I_1I_2\lambda_{12}^2\Big)^2}$$

$$= I_1\lambda_2\mu_1\frac{\Big((I_1+1)(I_2+1)\lambda_1\lambda_2 - I_1I_2\lambda_{12}^2\Big) - (I_2+1)\Big((I_1+1)\lambda_1\lambda_2 - I_1\lambda_{12}^2\Big)}{\Big((I_1+1)(I_2+1)\lambda_1\lambda_2 - I_1I_2\lambda_{12}^2\Big)^2},$$

$$= I_1\lambda_2\mu_1\frac{I_1\lambda_{12}^2}{\Big((I_1+1)(I_2+1)\lambda_1\lambda_2 - I_1I_2\lambda_{12}^2\Big)^2} > 0,$$

where the last inequality follows from the fact that the denominator of the fraction is strictly positive because of Lemma B.2 in Appendix B.

Part 2. The result follows from Part 1 and the expression for the profit of the ith investor exploiting the kth characteristic given in Equation (18).

Part 3. From Proposition 4.2 we have that the price-impact cost of the ith investor exploiting the first characteristic is $\zeta_{1id} = \mu_1\theta_{1id} - \lambda_1\theta_{1id}^2$. Therefore, the price impact from exploiting the first characteristic is

$$\zeta_{1id}/\theta_{1id} = \mu_1 - \lambda_1\theta_{1id}.$$

The result follows because Part 1 shows that θ_{1id} is increasing in I_2.

Part 4. The partial derivative of the aggregate investment position in the second characteristic with respect to I_2 is

$$\frac{\partial(\theta_{2d})}{\partial I_2} = \frac{\partial(I_2\theta_{2id})}{\partial I_2} = \theta_{2id} + I_2\frac{\partial(\theta_{2id})}{\partial I_2} \tag{D7}$$

$$= \theta_{2id}\left(1 - \frac{(I_1+1)\lambda_1\lambda_2 - I_1\lambda_{12}^2}{(I_1+1)\frac{I_2+1}{I_2}\lambda_1\lambda_2 - I_1\lambda_{12}^2}\right) < 0, \tag{D8}$$

where the inequality in (D8) holds because $\theta_{2id} < 0$ by Proposition 4.2 and the ratio inside the parenthesis is strictly smaller than one because of Lemma B.2 and the fact that $(I_2+1)/I_2 > 1$ for finite I_2.

Part 5. The partial derivative of the aggregate profit from the second characteristic with respect to I_2 is

$$\frac{\partial(I_2\pi_{2id})}{\partial I_2} = \pi_{2id} + I_2\frac{\partial\pi_{2id}}{\partial I_2} \tag{D9}$$

$$= \pi_{2id} + I_2 2\lambda_2\theta_{2id}\frac{\partial\theta_{2id}}{\partial I_2} \tag{D10}$$

$$= \pi_{2id} + I_2 2\lambda_2\theta_{2id}\frac{[(I_1+1)\lambda_1\lambda_2 - I_1\lambda_{12}^2]I_1\lambda_{12}\mu_1}{[(I_1+1)(I_2+1)\lambda_1\lambda_2 - I_1 I_2\lambda_{12}^2]^2} \tag{D11}$$

$$= \lambda_2\theta_{2id}^2\left(1 - 2\frac{(I_1+1)\lambda_1\lambda_2 - I_1\lambda_{12}^2}{(I_1+1)\frac{I_2+1}{I_2}\lambda_1\lambda_2 - I_1\lambda_{12}^2}\right). \tag{D12}$$

It suffices to show that the content of the parenthesis on the right-hand side of (D12) is negative, which holds if and only if

$$(I_1+1)\frac{I_2+1}{I_2}\lambda_1\lambda_2 - I_1\lambda_{12}^2 < 2(I_1+1)\lambda_1\lambda_2 - 2I_1\lambda_{12}^2$$

$$(I_1+1)\frac{I_2+1}{I_2}\lambda_1\lambda_2 < 2(I_1+1)\lambda_1\lambda_2 - I_1\lambda_{12}^2$$

$$(I_1+1)(I_2+1)\lambda_1\lambda_2 < 2(I_1+1)I_2\lambda_1\lambda_2 - I_1 I_2\lambda_{12}^2$$

$$(I_1+1)\lambda_1\lambda_2 < (I_1+1)I_2\lambda_1\lambda_2 - I_1 I_2\lambda_{12}^2,$$

which holds if and only if

$$I_2 > \frac{(I_1+1)\lambda_1\lambda_2}{(I_1+1)\lambda_1\lambda_2 - I_1\lambda_{12}^2}.$$

To see that the aggregate profit of the investors exploiting the second characteristic converges to zero as I_2 goes to infinity, note that

$$\pi_{2d} = I_2\pi_{2id} = I_2\lambda_2\theta_{2id}^2.$$

Then, it suffices to note that Equation (17) in Proposition 4.2 shows that the denominator of θ_{2id} grows linearly in I_2, and thus, the denominator of θ_{2id}^2 grows quadratically in I_2.

Proof of Proposition 4.6

Part 1. To show that the investment position in the first characteristic in the decentralized setting with $I_1 = 1$ is smaller than that of the centralized setting, we need to prove the following inequality:

$$\underbrace{\frac{2\lambda_2\mu_1}{4\lambda_1\lambda_2-\lambda_{12}^2}}_{\theta_{1d}} < \underbrace{\frac{\lambda_2\mu_1}{2(\lambda_1\lambda_2-\lambda_{12}^2)}}_{\theta_{1c}}, \tag{D13}$$

where the expression for the investment position in the first characteristic in the centralized setting, θ_{1c}, is obtained by setting the average return of the second characteristic equal to zero ($\mu_2 = 0$) in Equation (B13) of Proposition B.3 of Appendix B. Simplifying we have

$$\frac{1}{4\lambda_1\lambda_2-\lambda_{12}^2} < \frac{1}{4\lambda_1\lambda_2-4\lambda_{12}^2}.$$

By Lemma B.2, we know that the denominators of both the right- and left-hand sides of the inequality are strictly positive. Also, the denominator of the right-hand side term is smaller and thus (D13) holds.

Part 2. We now prove that the investment position in the second characteristic in the decentralized setting with $I_1 = 1$ is negative but higher than that in the centralized setting. Therefore, we prove the following inequalities:

$$0 > \underbrace{\frac{-\lambda_{12}\mu_1}{4\lambda_1\lambda_2-\lambda_{12}^2}}_{\theta_{2d}} > \underbrace{\frac{-\lambda_{12}\mu_1}{2(\lambda_1\lambda_2-\lambda_{12}^2)}}_{\theta_{2c}}, \tag{D14}$$

where the expression for the investment position in the second characteristic in the centralized setting, θ_{2c}, is obtained by setting the average return of the second characteristic equal to zero ($\mu_2 = 0$) in Equation (B14) of Proposition B.3. Note that the numerators of θ_{2d} and θ_{2c} are identical and negative, whereas the denominators of θ_{2d} and θ_{2c} are strictly positive by Lemma B.2. However, the denominator of θ_{2d} is larger than that of θ_{2c}, and thus, θ_{2d} is smaller in absolute value than θ_{2c}.

Part 3. From Equation (B16) in Proposition B.3, we know that the profits from the second characteristic in the centralized setting are zero for the case where Assumption 4.1(3) holds; that is, $\mu_2 = 0$. Moreover, Part 2 above and Equation (18) imply that the profits from the second characteristic in the decentralized setting are strictly positive.

Part 4. The total profits in the decentralized setting have to be smaller than those in the centralized setting because by Proposition B.3 we know that the optimal investment positions in the centralized setting are the unique maximizer to the total profit function.

Because we know from Part 3 that profits from the second characteristic are smaller in the centralized setting, and we have just shown that total profits are higher in the centralized setting, then we must have that profits from the first characteristic are larger in the centralized setting.

Proof of Lemma B.1

For the case with linear price impact given in Assumption 4.1(1), we have that $\alpha = 1$, and thus the price-impact at time t defined in Equation (2) becomes

$$\mathrm{PI}_t = \Lambda_t \Delta w_t, \tag{D15}$$

where the aggregate amount of trading is

$$\Delta w_t = \sum_{k=1}^{2} \sum_{i=1}^{I_k} \Delta w_{kit}, \tag{D16}$$

in which Δw_{kit} contains the portfolio-rebalancing trades for the ith investor in the kth characteristic:

$$\Delta w_{kit} = w_{kit}(\theta_{ki}) - w_{kit}^{+}(\theta_{ki}), \tag{D17}$$

$w_{kit}^{+}(\theta_{ki})$ is the portfolio of the ith investor in the kth characteristic before trading at time t:

$$w_{kit}^{+}(\theta_{ki}) = \theta_{ki}\ x_{k,t-1} \circ (e + r_t), \tag{D18}$$

e is the N-dimensional vector of ones, and $x \circ y$ is the componentwise (Hadamard) product of x and y. The price-impact cost at time t of the ith investor in the kth characteristic is:

$$\mathrm{PIC}_{kit} = \Delta w_{kit} \mathrm{PI}_t.$$

The lemma follows from straightforward algebra.

Proof of Lemma B.2

We first show the result for the empirically relevant case where there is a *discrete* joint probability distribution for the rebalancing-trade vectors, $\tilde{x}_{1t}$ and $\tilde{x}_{2t}$, and the price-impact matrix, Λ_t. By concatenating the rebalancing trades $\tilde{x}_{1t}$ and $\tilde{x}_{2t}$ and the matrices Λ_t for all realizations of the discrete distribution into panel vectors and matrices, it is straightforward to show that λ_1 and λ_2 are squared norms of certain vectors and λ_{12} is the scalar product of these same vectors. Therefore, it follows from the triangular inequality for norms that $\lambda_{12}^2 \leq \lambda_1\lambda_2$. Moreover, unless the rebalancing trades of the two characteristics are identical for every stock and every realization up to a change of scale, we have that the triangular inequality holds strictly $\lambda_{12}^2 < \lambda_1\lambda_2$. For the case where there is a continuous joint distribution for the rebalancing-trade vectors and the price-impact matrix, the result can be shown under mild assumptions by discretizing the continuous distribution and taking the limit when the granularity of the discretization goes to zero.

Proof of Proposition B.2

Part 1. By Lemma B.2 we know that $\lambda_k > 0$ for $k = 1, 2$ and thus, the decision problem of the ith investor in the kth characteristic is *strictly* concave. Therefore, there exists a unique global maximizer to the decision problem of the ith investor in the kth characteristic and it is given by the solution to the first-order optimality conditions:

$$2\lambda_1\theta_{1i} + \lambda_1\theta_{1,-i} + \lambda_{12}\sum_{j=1}^{I_2}\theta_{2j} = \mu_1, \quad \text{and} \tag{D19}$$

$$2\lambda_2\theta_{2i} + \lambda_2\theta_{2,-i} + \lambda_{12}\sum_{j=1}^{I_1}\theta_{1j} = \mu_2. \tag{D20}$$

Therefore, the investment positions θ_{1id} and θ_{2id} are a Nash equilibrium if and only if they satisfy the first-order optimality conditions of the investors in the first and second characteristics; that is, if they satisfy the following system of linear equations:

$$\begin{pmatrix} 2\lambda_1 & \lambda_1 & \cdots & \lambda_1 & \lambda_{12} & \lambda_{12} & \cdots & \lambda_{12} \\ \lambda_1 & 2\lambda_1 & \cdots & \lambda_1 & \lambda_{12} & \lambda_{12} & \cdots & \lambda_{12} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_1 & \lambda_1 & \cdots & 2\lambda_1 & \lambda_{12} & \lambda_{12} & \cdots & \lambda_{12} \\ \lambda_{12} & \lambda_{12} & \cdots & \lambda_{12} & 2\lambda_2 & \lambda_2 & \cdots & \lambda_2 \\ \lambda_{12} & \lambda_{12} & \cdots & \lambda_{12} & \lambda_2 & 2\lambda_2 & \cdots & \lambda_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_{12} & \lambda_{12} & \cdots & \lambda_{12} & \lambda_2 & \lambda_2 & \cdots & 2\lambda_2 \end{pmatrix} \begin{pmatrix} \theta_{11d} \\ \theta_{12d} \\ \vdots \\ \theta_{1I_1d} \\ \theta_{21d} \\ \theta_{22d} \\ \vdots \\ \theta_{2I_2d} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_1 \\ \vdots \\ \mu_1 \\ \mu_2 \\ \mu_2 \\ \vdots \\ \mu_2 \end{pmatrix}. \tag{D21}$$

We now prove that there is a unique Nash equilibrium by showing that the matrix on the left hand side of (D21) is nonsingular. Assume by contradiction that there is a nonzero vector of θ_{1i}'s and θ_{2i}'s that satisfies the following:

$$\begin{pmatrix} 2\lambda_1 & \lambda_1 & \cdots & \lambda_1 & \lambda_{12} & \lambda_{12} & \cdots & \lambda_{12} \\ \lambda_1 & 2\lambda_1 & \cdots & \lambda_1 & \lambda_{12} & \lambda_{12} & \cdots & \lambda_{12} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_1 & \lambda_1 & \cdots & 2\lambda_1 & \lambda_{12} & \lambda_{12} & \cdots & \lambda_{12} \\ \lambda_{12} & \lambda_{12} & \cdots & \lambda_{12} & 2\lambda_2 & \lambda_2 & \cdots & \lambda_2 \\ \lambda_{12} & \lambda_{12} & \cdots & \lambda_{12} & \lambda_2 & 2\lambda_2 & \cdots & \lambda_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_{12} & \lambda_{12} & \cdots & \lambda_{12} & \lambda_2 & \lambda_2 & \cdots & 2\lambda_2 \end{pmatrix} \begin{pmatrix} \theta_{11} \\ \theta_{12} \\ \vdots \\ \theta_{1I_1} \\ \theta_{21} \\ \theta_{22} \\ \vdots \\ \theta_{2I_2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}. \qquad \text{(D22)}$$

Then, any solution to (D22) must satisfy the first I_1 equations in (D22), which can be rewritten as

$$\begin{pmatrix} 2\lambda_1 & \lambda_1 & \cdots & \lambda_1 \\ \lambda_1 & 2\lambda_1 & \cdots & \lambda_1 \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_1 & \lambda_1 & \cdots & 2\lambda_1 \end{pmatrix} \begin{pmatrix} \theta_{11} \\ \theta_{12} \\ \vdots \\ \theta_{1I_1} \end{pmatrix} = -\lambda_{12} \sum_{i=1}^{I_2} \theta_{2i} e, \qquad \text{(D23)}$$

where e is the I_1-dimensional vector of ones. The matrix on the left-hand side of (D23) is nonsingular because by Lemma B.2 we know that $\lambda_1 > 0$. Moreover, this matrix is symmetric with respect to the I_1 investors in the first characteristic. Therefore, any solution to Equation (D23) must be symmetric with respect to the I_1 investors in the first characteristic; that is, $\theta_{1i} = \theta_1$ for $i = 1, 2, \ldots, I_1$. Consequently Equation (D22) can be rewritten as

$$\begin{pmatrix} (I_1+1)\lambda_1 & \lambda_{12} & \lambda_{12} & \cdots & \lambda_{12} \\ I_1\lambda_{12} & 2\lambda_2 & \lambda_2 & \cdots & \lambda_2 \\ I_1\lambda_{12} & \lambda_2 & 2\lambda_2 & \cdots & \lambda_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ I_1\lambda_{12} & \lambda_2 & \lambda_2 & \cdots & 2\lambda_2 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_{21} \\ \theta_{22} \\ \vdots \\ \theta_{2I_2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}. \qquad \text{(D24)}$$

Using similar arguments as above, it is straightforward to show that any solution to Equation (D24) must be symmetric with respect to the I_2 investors in the second characteristic; that is, $\theta_{2d} = \theta_2$ for $i = 1, 2, \ldots, I_2$. Thus, we can express (D22) as follows

$$\begin{pmatrix} (I_1+1)\lambda_1 & I_2\lambda_{12} \\ I_1\lambda_{12} & (I_2+1)\lambda_2 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \qquad \text{(D25)}$$

The matrix on the left-hand side of (D25) is nonsingular for any I_1 and I_2 different from zero because its determinant is $(I_1+1)(I_2+1)\lambda_1\lambda_2 - I_1 I_2 \lambda_{12}^2$, which is nonzero by Lemma B.2.

Consequently, there is a unique Nash equilibrium given by the unique solution to the linear system of equations in (D21).

Part 2. By arguments similar to those in Part 1, any solution to (D21) must be symmetric with respect to the I_1 investors in the first characteristic and with respect to the I_2 investors in the second characteristic; that is, $\theta_{kid} = \theta_{kd}$ for $i = 1, 2, \ldots, I_k$ and $k = 1, 2$.

Part 3. Therefore, the unique equilibrium is the solution to the following system of two linear equations with two variables

$$\begin{pmatrix} (I_1+1)\lambda_1 & I_2\lambda_{12} \\ I_1\lambda_{12} & (I_2+1)\lambda_2 \end{pmatrix} \begin{pmatrix} \theta_{1d} \\ \theta_{2d} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}. \tag{D26}$$

The above system of two equations can be solved by premultiplying the vector of characteristic means by the inverse of the left-hand side matrix. This gives the following optimal solutions:

$$\theta_{1id} = \frac{(I_2+1)\,\lambda_2\mu_1 - I_2\lambda_{12}\mu_2}{(I_1+1)(I_2+1)\lambda_1\lambda_2 - I_1 I_2 \lambda_{12}^2},$$

$$\theta_{2id} = \frac{(I_1+1)\lambda_1\mu_2 - I_1\lambda_{12}\mu_1}{(I_1+1)(I_2+1)\lambda_1\lambda_2 - I_1 I_2 \lambda_{12}^2}.$$

Part 4. The profit of the ith investor in the kth characteristic is her expected return net of price impact multiplied by her investment position. Therefore, it suffices to show that the expected return net of price impact of the ith investor in the kth characteristic is $\bar{\mu}_{kid} = \lambda_k\theta_{kid}$. The expected return net of price impact of the ith investor in the first characteristic is

$$\bar{\mu}_{1id} = \mu_1 - \lambda_1 I_1 \theta_{1id} - \lambda_{12} I_2 \theta_{2id}.$$

Now, using the ith investor's first-order conditions, we have that:

$$0 = \mu_1 - \lambda_1(I_1+1)\theta_{1id} - \lambda_{12} I_2 \theta_{2id}.$$

Therefore, substituting the last equation into the expression for $\bar{\mu}_{1id}$, we obtain $\bar{\mu}_{1id} = \lambda_1\theta_{1id}$. The result for the ith investor in the second characteristic is obtained similarly.

Part 5. The result holds because the investor's price-impact cost is the difference between the investor's gross return ($\mu_k\theta_{kid}$) and the investor's profit ($\lambda_k\theta_{kid}^2$) given in Part 4.

Part 6. The result follows from Part 5 by noting that the price impact experienced by the ith investor exploiting the kth characteristic is equal to her price-impact cost divided by her investment position.

Proof of Proposition B.3

Part 1. The decision in the centralized setting is given in (B7). By Lemma B.2 we have that $\lambda_1\lambda_2 > \lambda_{12}^2$ and therefore the decision problem in the centralized setting is strictly concave and there exists a unique maximizer.

Parts 2, 3, and 4. The unique maximizer is given by the first-order optimality conditions for the single investor in the centralized setting:

$$\begin{pmatrix} 2\lambda_1 & 2\lambda_{12} \\ 2\lambda_{12} & 2\lambda_2 \end{pmatrix} \begin{pmatrix} \theta_{1c} \\ \theta_{2c} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}. \tag{D27}$$

The result then follows from straightforward algebra.

References

Almgren, Robert, Chee Thum, Emmanuel Hauptmann, and Hong Li, 2005, Direct estimation of equity market impact, *Risk* 18, 58–62.

An, Li, Shiyang Huang, Dong Lou, and Jiahong Shi, 2021, An anatomy of long-short equity funds, available at SSRN 3813790.

Baert, Rick, 2018, Growth of ETFs reflects passive shift; three largest firms hold 79% of assets, *Pensions & Investments* (May 28).

Barroso, Pedro, 2012, Three essays in financial markets, PhD dissertation at Universidade Nova de Lisboa.

Barroso, Pedro, Roger M. Edelen, and Paul Karehnke, 2021, Crowding and tail risk in momentum returns, *Journal of Financial and Quantitative Analysis* 57, 1313–1342.

Barroso, Pedro, and Pedro Santa-Clara, 2015, Beyond the carry trade: Optimal currency portfolios, *Journal of Financial and Quantitative Analysis* 50, 1037–1056.

Berk, Jonathan B., and Richard C. Green, 2004, Mutual fund flows and performance in rational markets, *Journal of Political Economy* 112, 1269–1295.

Berk, Jonathan B., and Jules H. van Binsbergen, 2017, Mutual funds in equilibrium, *Annual Review of Financial Economics* 9, 147–167.

BlackRock, 2021, What is Factor Investing?, `https://www.blackrock.com/us/individual/investment-ideas/what-is-factor-investing`.

Bonelli, Maxime, Augustin Landier, Guillaume Simon, and David Thesmar, 2019, The capacity of trading strategies, available at SSRN 2585399.

Brandt, Michael W., Pedro Santa-Clara, and Rossen Valkanov, 2009, Parametric portfolio policies: Exploiting characteristics in the cross-section of equity returns, *Review of Financial Studies* 22, 3411–3447.

Brown, Gregory W., Philip Howard, and Christian T. Lundblad, 2020, Crowded trades and tail risk, forthcoming in *Review of Financial Studies*.

Carlson, Debbie, 2019, Invesco focusing on scale, *ETF.com* (January 24).

Chen, Andrew, and Mihail Velikov, 2021, Zeroing in on the expected returns of anomalies, forthcoming in *Journal of Financial and Quantative Analysis*.

Chincarini, Ludwig B., 2017, Transaction costs and crowding, *Quantitative Finance* 18, 1389–1410.

Cournot, Antoine-Augustin, 1838, *Recherches sur les Principes Mathématiques de la Théorie des Richesses par Augustin Cournot* (chez L. Hachette, Paris).

Dasgupta, Amil, Andrea Prat, and Michela Verardo, 2011, Institutional trade persistence and long-term equity returns, *Journal of Finance* 66, 635–653.

DeMiguel, Victor, Alberto Martin-Utrera, Francisco J. Nogales, and Raman Uppal, 2020, A transaction-cost perspective on the multitude of firm characteristics, *Review of Financial Studies* 33, 2180–2222.

Detzel, Andrew, Robert Novy-Marx, and Mihail Velikov, 2021, Model selection with transaction costs, available at SSRN 3088828.

Doshi, Hitesh, Redouane Elkamhi, and Mikhail Simutin, 2015, Managerial activeness and mutual fund performance, *Review of Asset Pricing Studies* 5, 156–184.

Drechsler, Itamar, Alan Moreira, and Alexi Savov, 2020, Liquidity and volatility, available at SSRN 3133291.

Edelen, Roger M., Richard B. Evans, and Gregory B. Kadlec, 2007, Scale effects in mutual fund performance: The role of trading costs, available at SSRN 951367.

Feldman, David, Konark Saxena, and Jingrui Xu, 2020, Is the active fund management industry concentrated enough?, *Journal of Financial Economics* 136, 23–43.

Feldman, David, Konark Saxena, and Jingrui Xu, 2021, One global village? Competition in the international active fund management industry, available at SSRN 3330131.

Flood, Chris, 2019, Investors' smart money piles into smart beta ETFs, *Financial Times* (February 11).

Franzoni, Francesco A., Alberto Plazzi, and Efe Cotelioglu, 2021, What constrains liquidity provision? Evidence from hedge fund trades, *Review of Finance* 25, 485–517.

Frazzini, Andrea, Ronen Israel, and Tobias J. Moskowitz, 2015, Trading costs of asset pricing anomalies, Fama-Miller Working Paper.

Frazzini, Andrea, Ronen Israel, and Tobias J. Moskowitz, 2018, Trading costs, AQR Working Paper.

Green, Jeremiah, John R. M. Hand, and X. Frank Zhang, 2017, The characteristics that provide independent information about average U.S. monthly stock returns, *Review of Financial Studies* 30, 4389–4436.

Grinold, Richard C., and Ronald N. Kahn, 2000, *Active portfolio management* (McGraw-Hill, New York), 2nd edition.

Gârleanu, Nicolae, and Lasse H. Pedersen, 2013, Dynamic trading with predictable returns and transaction costs, *Journal of Finance* 68, 2309–2340.

Harvey, Campbell R., Yan Liu, Eric K. M. Tan, and Min Zhu, 2020, Crowding: Evidence from fund managerial structure, available at SSRN 3554636.

Hoberg, Gerard, Nitin Kumar, and Nagpurnanand Prabhala, 2018, Mutual fund competition, managerial skill, and alpha persistence, *Review of Financial Studies* 31, 1896–1929.

Hoberg, Gerard, Nitin Kumar, and Nagpurnanand Prabhala, 2020, Buy-side competition and momentum profits, forthcoming in *Review of Financial Studies*.

Huang, Shiyang, Yang Song, and Hong Xiang, 2021, Noise trading and asset pricing factors, available at SSRN 3359356.

Jacobs, Bruce I., and Kenneth N. Levy, 2014, Investing in a multidimensional market, *Financial Analysts Journal* 70, 6–12.

Khandani, Amir E., and Andrew W. Lo, 2011, What happened to the quants in August 2007? Evidence from factors and transactions data, *Journal of Financial Markets* 14, 1–46.

Korajczyk, Robert A., and Ronnie Sadka, 2004, Are momentum profits robust to trading costs?, *Journal of Finance* 59, 1039–1082.

Lesmond, David A., Michael J. Schill, and Chunsheng Zhou, 2004, The illusory nature of momentum profits, *Journal of Financial Economics* 71, 349–380.

Li, Jiacui, 2021, What drives the size and value factors?, available at SSRN 2909960.

Lou, Dong, 2012, A flow-based explanation for return predictability, *Review of Financial Studies* 25, 3457–3489.

Lou, Dong, and Christopher Polk, 2022, Comomentum: Inferring arbitrage activity from return correlations, *Review of Financial Studies* 35, 3272–3302.

Nagel, Stefan, 2012, Evaporating liquidity, *Review of Financial Studies* 25, 2005–2039.

Novy-Marx, Robert, and Mihail Velikov, 2016, A taxonomy of anomalies and their trading costs, *Review of Financial Studies* 29, 104–147.

Pástor, Ľuboš, and Robert F. Stambaugh, 2012, On the size of the active management industry, *Journal of Political Economy* 120, 740–781.

Pástor, Ľuboš, Robert F. Stambaugh, and Lucian A. Taylor, 2015, Scale and skill in active management, *Journal of Financial Economics* 116, 23–45.

Ratcliffe, Ronald, Paolo Miranda, and Andrew Ang, 2017, Capacity of smart beta strategies: A transaction cost perspective, *Journal of Index Investing* 8, 39–50.

Rhoades, Stephen A., 1993, The herfindahl-hirschman index, *Federal Reserve Bulletin* 79, 188–189.

Stein, Jeremy C., 2009, Presidential address: Sophisticated investors and market efficiency, *Journal of Finance* 64, 1517–1548.

Torre, Nicolo G., and Mark J. Ferrari, 1997, *Market Impact Model Handbook* (BARRA Inc, Berkeley).

van Binsbergen, Jules H., Martijn Boons, Christian Opp, and Andrea Tamoni, 2021, Dynamic asset (mis) pricing: Build-up vs. resolution anomalies, CEPR Discussion Papers.

Wahal, Sunil, and Albert Y. Wang, 2011, Competition among mutual funds, *Journal of Financial Economics* 99, 40–59.

Wermers, Russ, 1999, Mutual fund herding and the impact on stock prices, *Journal of Finance* 54, 581–622.

Winkelbauer, Andreas, 2012, Moments and absolute moments of the normal distribution, arXiv preprint 1209.4340.

Internet Appendix to

Can Competition Increase Profits in Factor Investing?

This Internet Appendix contains several robustness checks. Section IA.1 checks the robustness of our theoretical findings to considering risk-averse investors. Section IA.2 checks the robustness of our empirical results regarding trading diversification to considering two subsamples of the data. Section IA.3 checks the robustness of the empirical calibration of the game-theoretic model to considering three different pairs of characteristics: (i) "book to market" and "gross profitability," (ii) "momentum" and "gross profitability," and (iii) "momentum" and "book to market." Section IA.4 checks the robustness of our findings to considering persistent price-impact costs.

IA.1 Game-theoretic model with risk-averse investors

In the main body of the manuscript, we consider risk-neutral investors. We now extend the model to consider risk-averse investors. We assume that the investors' absolute risk-aversion parameters in the decentralized setting increase with the number of competitors. In particular, we assume that the absolute risk-aversion parameters of the investors in the first and second characteristics are $\gamma_1 = (I_1 + 1)/(2\overline{\gamma}_1)$ and $\gamma_2 = (I_2 + 1)/(2\overline{\gamma}_2)$, respectively, where I_1 and I_2 are the number of investors exploiting the first and second characteristics, respectively, and $\overline{\gamma}_1$ and $\overline{\gamma}_2$ are constants. This assumption greatly simplifies the analysis, but it is also reasonable because each investor makes a smaller investment as the number of competitors increases, and hence the investor's absolute risk aversion must increase with the number of competitors.

The ith investor in the first characteristic chooses her investment position θ_{1i} to optimize her mean-variance utility net of price-impact costs

$$\max_{\theta_{1i}} \quad \theta_{1i}\mu_1 - \frac{\gamma_1}{2}\theta_{1i}\sigma_1^2\theta_{1i} - \theta_{1i}\lambda_1(\theta_{1i} + \theta_{1,-i}) - \theta_{1i}\lambda_{12}\sum_{j=1}^{I_2}\theta_{2j}, \tag{IA1}$$

where σ_1^2 is the variance of the first characteristic return. Similarly, the decision problem of the ith investor in the second characteristic is

$$\max_{\theta_{2i}} \quad \theta_{2i}\mu_2 - \frac{\gamma_2}{2}\theta_{2i}\sigma_2^2\theta_{2i} - \theta_{2i}\lambda_2(\theta_{2i} + \theta_{2,-i}) - \theta_{2i}\lambda_{12}\sum_{j=1}^{I_1}\theta_{1j}, \tag{IA2}$$

where σ_2^2 is the variance of the second characteristic return. Using similar arguments to those in the proofs of Propositions B.2 and B.3, the equilibrium is symmetric and thus the optimality condition of the ith investor in the first characteristic can be written as

$$\gamma_1\sigma_1^2\theta_{1i} + (I_1 + 1)\lambda_1\theta_{1i} + I_2\lambda_{12}\theta_{2j} - \mu_1 = 0, \tag{IA3}$$

which can be rewriten as

$$(I_1+1)\left(\frac{\overline{\gamma}_1}{2}\sigma_1^2+\lambda_1\right)\theta_{1i}+I_2\lambda_{12}\theta_{2j}-\mu_1=0. \tag{IA4}$$

Similarly, the optimality condition for the ith investor in the second characteristic is

$$(I_2+1)\left(\frac{\overline{\gamma}_2}{2}\sigma_2^2+\lambda_2\right)\theta_{2i}+I_1\lambda_{12}\theta_{1j}-\mu_2=0. \tag{IA5}$$

From these optimality conditions, it is straightforward to show that the equilibrium quantities in the case with risk-averse investors are those given in Propositions B.2 and B.3 after replacing the transaction cost parameters λ_1 and λ_2 with $\widetilde{\lambda}_1=\frac{\overline{\gamma}_1}{2}\sigma_1^2+\lambda_1$ and $\widetilde{\lambda}_2=\frac{\overline{\gamma}_2}{2}\sigma_2^2+\lambda_2$, respectively. Therefore, the results in the main body of the manuscript continue to hold for the case with risk-averse investors.

IA.2 Trading diversification with a single investor: subsample analysis

We now study the robustness of our empirical findings regarding the effect of trading diversification to considering two subsamples covering the first and second half of our dataset. Table 2 in the main body of the manuscript shows that trading diversification leads to an increase in total capacity of 45%, total investment of 43%, and profit of 22%. Tables IA.1 and IA.2 report the results for the first and second halves of our sample, respectively. We observe that our results are robust to considering these subsamples. In particular, trading diversification in the first half of the sample leads to an increase in total capacity of 51%, total investment of 54%, and profit of 54%, and in the second half of the sample to an increase in total capacity of 40%, total investment of 39%, and profit of 19%. Thus, for both subsamples trading diversification has a substantial effect on the equilibrium quantities.[25]

Comparing the benefits of trading diversification for the two subsamples, we observe that they are slightly larger for the first subsample. The reason for this is that stock-trading volumes are relatively smaller for the first subsample, and thus the price-impact costs given by Equation (13) are more important in the first subsample.[26] Nonetheless, trading diversification remains important also in the second subsample causing capacity and investment to increase by around 40% and profit to increase by almost a fifth.

[25]Note that when considering the 18 characteristics in combination in the second subsample, it is optimal to assign a negative weight of −\$0.687 billion to the "chatoia" characteristic. Although this negative weight on "chatoia" makes a negative contribution to profit of −\$0.48 million, it is more than compensated by the reduction in the price-impact costs of the other characteristics, and thus, increase in their profit contribution, because of trading diversification.

[26]Note that although we report capacity, investment, and profits in terms of market capitalization at the end of our full sample (December 2018), daily trading volumes have grown faster than market capitalization in our sample. For instance, the median stock daily trading volume has grown by a factor of 48.49 in our

IA.3 Empirical calibration of game-theoretic model

In Section 4.5, we calibrate the game-theoretic model for the case with "investment (asset growth)" as the first characteristic and "gross profitability" as the second. We now study the robustness of our main findings to considering instead the following three different pairs of characteristics: (i) "book to market" and "gross profitability," (ii) "momentum" and "gross profitability," and (iii) "momentum" and "book to market."

Figures IA.1 and IA.2 depict the investment position and profits for the case with "book to market" as the first characteristic and "gross profitability" as the second. The figures show that our main findings are robust. Figure IA.1 shows that an increase in competition to exploit "book to market" erodes its profits. Figure IA.2 shows that an increase in competition to exploit "gross profitability" increases the optimal investment position and profits from "book to market" because of the positive externality generated by trading diversification across the two characteristics.

Figures IA.3 and IA.4 depict the investment position and profits for the case with "momentum" as the first characteristic and "gross profitability" as the second. Consistent with the results in Table 2, Figures IA.3 and IA.4 show that the optimal investment positions in the momentum characteristic are below $1 billion. Nonetheless, the figures show that our main findings are again robust. Figure IA.3 shows that an increase in competition to exploit "momentum" erodes its profits. Figure IA.4 shows that an increase in competition to exploit "gross profitability" increases the optimal investment position and profits from "momentum" because of the positive externality generated by trading diversification across the two characteristics.

Finally, Figure IA.5 depicts the investment position and profits for the case with "momentum" as the first characteristic and "book to market" as the second. The figure shows again that an increase in competition to exploit "book to market" increases the optimal investment position and profits from "momentum" because of the positive externality generated by trading diversification across the two characteristics.

IA.4 Persistent price-impact costs

In the main body of the manuscript, we consider the case where trading a characteristic exerts only temporary price impact, which is the case considered in the empirical literature on the capacity of quantitative strategies (Korajczyk and Sadka, 2004; Lesmond et al., 2004;

sample, whereas the median stock market capitalization has grown by a factor of 37.95. As a result, even though quantities are reported in terms of market capitalization at the end of our full sample, price-impact costs are relatively more important in the first subsample.

Novy-Marx and Velikov, 2016; Ratcliffe et al., 2017; Frazzini et al., 2018). If a characteristic reflects compensation for risk, then trading the characteristic will exert mainly temporary price impact, but if it reflects mispricing, then trading it may exert *both* temporary and permanent price impact. In this section, we extend our game-theoretic model presented in Section 4 to account for *persistent* price-impact costs, which include as particular cases both permanent and temporary price-impact costs. We show that our findings are robust to considering the more general case with persistent price-impact costs.

We consider the model of persistent price impact by Gârleanu and Pedersen (2013), but stated in terms of return distortions instead of price distortions. Like Gârleanu and Pedersen (2013), we assume that the amount traded has a linear effect on prices. In addition, we assume the following dynamics for the persistent posttrade return distortion:

$$d_{t+1} = (I_N - \Phi)\left(d_t + C\Delta w_t\right), \tag{IA6}$$

where d_t is the persistent return distortion at time t, C is the *persistent* price-impact cost matrix, Δw_t is the aggregate trade vector defined in (4), Φ is a diagonal matrix of mean-reversion parameters that determines how fast persistent return distortions revert to zero, and I_N is the N-dimensional identity matrix. For instance, when $\Phi = 0$, persistent return distortions do not mean revert, and thus they reflect permanent price impact, but when $\Phi = I_N$ then return distortions revert in a single period, and thus they reflect temporary price impact. The expected return due to posttrade persistent return distortion is:

$$\mu^d_{t+1} = -\Phi\left(d_t + C\Delta w_t\right). \tag{IA7}$$

Applying (IA6) recursively to (IA7), we can rewrite the expected return due to posttrade persistent return distortion at time $t+1$ as:

$$\mu^d_{t+1} = -\Phi \sum_{\tau=0}^{t-2} (I_N - \Phi)^\tau C\Delta w_{t-\tau}. \tag{IA8}$$

Now, using the definition of Δw_t in (4) for the case with $K = 2$ characteristics, we have that

$$\mu^d_{t+1} = -\Phi \sum_{\tau=0}^{t-2} (I_N - \Phi)^\tau C(\theta_1\, \tilde{x}_{1,t-\tau} + \theta_2\, \tilde{x}_{2,t-\tau}). \tag{IA9}$$

Given a sample of T observations, we estimate the unconditional expectation of the portfolio return due to posttrade persistent return distortion as

$$\mu^p = -\frac{1}{T-2}\sum_{t=2}^{T-1}(\theta_1\, x_{1t} + \theta_2\, x_{2t})^\top \Phi \sum_{\tau=0}^{t-2}(I_N - \Phi)^\tau C(\theta_1\, \tilde{x}_{1,t-\tau} + \theta_2\, \tilde{x}_{2,t-\tau}) \tag{IA10}$$

$$= -\theta_1\mu^p_1\theta_1 - \theta_2\mu^p_2\theta_2 - \theta_1(\mu^p_{12} + \mu^p_{21})\theta_2, \tag{IA11}$$

where

$$\mu_1^p = \frac{1}{T-2}\sum_{t=2}^{T-1} x_{1t}^\top \Phi \sum_{\tau=0}^{t-2}(I_N - \Phi)^\tau C\tilde{x}_{1,t-\tau}, \tag{IA12}$$

$$\mu_2^p = \frac{1}{T-2}\sum_{t=2}^{T-1} x_{2t}^\top \Phi \sum_{\tau=0}^{t-2}(I_N - \Phi)^\tau C\tilde{x}_{2,t-\tau}, \tag{IA13}$$

$$\mu_{12}^p = \frac{1}{T-2}\sum_{t=2}^{T-1} x_{1t}^\top \Phi \sum_{\tau=0}^{t-2}(I_N - \Phi)^\tau C\tilde{x}_{2,t-\tau}, \quad \text{and} \tag{IA14}$$

$$\mu_{21}^p = \frac{1}{T-2}\sum_{t=2}^{T-1} x_{2t}^\top \Phi \sum_{\tau=0}^{t-2}(I_N - \Phi)^\tau C\tilde{x}_{1,t-\tau} \tag{IA15}$$

are the unconditional expectations of the portfolio returns due to persistent posttrade return distortions of the first characteristic, the second characteristic, the first characteristic on the second characteristic, and the second characteristic on the first characteristic, respectively.

In addition to the return due to persistent posttrade return distortion, we consider the following marked-to-market portfolio return:

$$\text{MMR}(\Delta w_t) = -w_{t-1}^{+\top} C\Delta w_t - \frac{1}{2}\Delta w_t^\top C\Delta w_t, \tag{IA16}$$

where the first term on the right-hand side of (IA16) is the marked-to-market return from the portfolio before trading at time t, w_{t-1}^+, due to the return distortion induced by the rebalancing trade Δw_t, and the second term is the marked-to-market return from the rebalancing trade, Δw_t, which is executed at an average return distortion of $C\Delta w_t/2$. Given T observations and using the definition of Δw_t in (4) for the case with $K = 2$ characteristics, we have that the unconditional expectation of the marked-to-market portfolio return is:

$$\begin{aligned}
\text{MMR} = &-\frac{1}{T-2}\sum_{t=2}^{T-1}[(\theta_1\, x_{1t-1}^+ + \theta_2\, x_{2t-1}^+)^\top C(\theta_1\, \tilde{x}_{1t} + \theta_2\, \tilde{x}_{2t}) \\
&+ \frac{1}{2}(\theta_1\, \tilde{x}_{1t} + \theta_2\, \tilde{x}_{2t})^\top C(\theta_1\, \tilde{x}_{1t} + \theta_2\, \tilde{x}_{2t})], \\
&= -\theta_1\lambda_1^{p1}\theta_1 - \theta_2\lambda_2^{p1}\theta_2 - \theta_1(\lambda_{12}^{p1} + \lambda_{21}^{p1})\theta_2 - \theta_1\lambda_1^{p2}\theta_1 - \theta_2\lambda_2^{p2}\theta_2 - 2\theta_1\lambda_{12}^{p2}\theta_2 \\
&= -\theta_1(\lambda_1^{p1} + \lambda_1^{p2})\theta_1 - \theta_2(\lambda_2^{p1} + \lambda_2^{p2})\theta_2 - \theta_1(\lambda_{12}^{p1} + \lambda_{21}^{p1} + 2\lambda_{12}^{p2})\theta_2,
\end{aligned} \tag{IA17}$$

where

$$\lambda_1^{p1} = \frac{1}{T-2}\sum_{t=2}^{T-1} x_{1t-1}^{+\top} C\tilde{x}_{1t}, \quad \lambda_2^{p1} = \frac{1}{T-2}\sum_{t=2}^{T-1} x_{2t-1}^{+\top} C\tilde{x}_{2t}, \quad \lambda_{12}^{p1} = \frac{1}{T-2}\sum_{t=2}^{T-1} x_{1t-1}^{+\top} C\tilde{x}_{2t}, \quad \text{and}$$

$$\lambda_{21}^{p1} = \frac{1}{T-2}\sum_{t=2}^{T-1} x_{2t-1}^{+\top} C\tilde{x}_{1t}$$

are the unconditional expectations of the marked-to-market returns of: the first characteristic portfolio before rebalancing at time t, the second characteristic portfolio before rebalancing at time t, the first characteristic portfolio before rebalancing at time t induced by the rebalancing trade on the second characteristic, and the second characteristic portfolio before rebalancing at time t induced by the rebalancing trade on the first, respectively, and

$$\lambda_1^{p2} = \frac{1}{2(T-2)} \sum_{t=2}^{T-1} \tilde{x}_{1t}^\top C \tilde{x}_{1t}, \quad \lambda_2^{p2} = \frac{1}{2(T-2)} \sum_{t=2}^{T-1} \tilde{x}_{2t}^\top C \tilde{x}_{2t}, \quad \text{and} \quad \lambda_{12}^{p2} = \frac{1}{2(T-2)} \sum_{t=2}^{T-1} \tilde{x}_{1t}^\top C \tilde{x}_{2t}$$

are the unconditional expectations of the marked-to-market returns from the first-characteristic rebalancing trade, the second-characteristic rebalancing trade, and the interaction between the first- and second-characteristic rebalancing trades, respectively.

Adding the unconditional expectations of the portfolio return due to return distortion (IA11) and the marked-to-market return (IA17), we obtain the persistent price-impact costs:

$$\text{PPIC} = \theta_1(\mu_1^p - \lambda_1^{p1} - \lambda_1^{p2})\theta_1 + \theta_2(\mu_2^p - \lambda_2^{p1} - \lambda_2^{p2})\theta_2 + \theta_1(\mu_{12}^p + \mu_{21}^p - \lambda_{12}^{p1} - \lambda_{21}^{p1} - 2\lambda_{12}^{p2})\theta_2.$$

The following proposition shows that when the matrix of mean-reversion parameters converges to the identity matrix, the model of persistent price-impact costs introduced in this appendix converges to the model of temporary quadratic price-impact cost considered in the main body of the manuscript.

Proposition IA.4.1 *Let $C = 2\Lambda$, where Λ is the diagonal price-impact cost matrix defined in* (3)*. Then, as the persistent price-impact cost matrix Φ in* (IA7) *converges to the identity matrix, the model of persistent price-impact costs converges to the model of temporary price-impact costs introduced in Lemma B.1.*

Proof of Proposition IA.4.1: Let the diagonal elements of matrix Φ in (IA7) converge to one; that is, $\Phi_{i,i} \to 1$. Then, from (IA8) we have that the excess return due to posttrade price distortion at time $t+1$ is:

$$\mu_{t+1}^d = -\Phi C \Delta w_t. \tag{IA18}$$

Accordingly, given a sample of T observations and assuming that $\Phi \to I_N$, we can estimate the unconditional expected portfolio return due to price distortions as

$$\mu^p = -\frac{1}{T-2} \sum_{t=2}^{T-1} w_t^\top \Phi C \Delta w_t, \tag{IA19}$$

which can be rewritten by noting that $w_t = w_{t-1}^+ + \Delta w_t$. Thus, we have that when $\Phi \to I_N$, the global persistent price-impact costs can be written as:

$$\text{GPPIC} = \frac{1}{T-2} \sum_{t=2}^{T-1} \left[(w_{t-1}^+ + \Delta w_t)^\top C \Delta w_t - w_{t-1}^+ C \Delta w_t - \frac{1}{2} \Delta w_t^\top C \Delta w_t \right], \tag{IA20}$$

which can be simplified as follows:

$$\text{PPIC} = \frac{1}{T-2}\sum_{t=2}^{T-1}\left[\frac{1}{2}\Delta w_t^\top C \Delta w_t\right]. \tag{IA21}$$

Therefore, if $C = 2\Lambda$, equation (IA21) coincides with the definition of price-impact costs in (7) for the case of linear price-impact costs when $\alpha = 1$. ■

We now state the decisions of the investors in the first and second characteristics in the decentralized setting, and the decision in the centralized setting. The ith investor in the first characteristic chooses her investment position θ_{1i} to maximize the difference between her persistent price-impact cost and expected return:

$$\max_{\theta_{1i}} \quad \theta_{1i}\mu_1 - \theta_{1i}\widetilde{\lambda}_1\theta_{1i} - \theta_{1i}\widetilde{\lambda}_1\sum_{j\neq i}^{I_1}\theta_{1,j} - \theta_{1i}\widetilde{\lambda}_{12}\sum_{j=1}^{I_2}\theta_{2j}, \tag{IA22}$$

where $\widetilde{\lambda}_1 = \mu_1^p - \lambda_1^{p1} - \lambda_1^{p2}$ is the persistent price-impact cost from trading the first characteristic, and $\widetilde{\lambda}_{12} = \mu_{12}^p - \lambda_{12}^{p1} - \lambda_{12}^{p2}$ is the persistent price-impact costs from the interaction between the first and second characteristics.

Similarly, the decision of the ith investor in the second characteristic is

$$\max_{\theta_{2i}} \quad \theta_{2i}\mu_2 - \theta_{2i}\widetilde{\lambda}_2\theta_{2i} - \theta_{2i}\widetilde{\lambda}_2\sum_{j\neq i}^{I_2}\theta_{2,j} - \theta_{2i}\widetilde{\lambda}_{12}\sum_{j=1}^{I_1}\theta_{1j}, \tag{IA23}$$

where $\widetilde{\lambda}_2 = \mu_2^p - \lambda_2^{p1} - \lambda_2^{p2}$ is the persistent price-impact cost from trading the second characteristic, and $\widetilde{\lambda}_{21} = \mu_{21}^p - \lambda_{21}^{p1} - \lambda_{12}^{p2}$ is the persistent price-impact cost from the interaction between the two characteristics.

Finally, the decision in the centralized setting is:

$$\max_{\theta_{1c},\theta_{2c}} \quad \theta_{1c}\mu_1 + \theta_{2c}\mu_2 - \theta_{1c}\widetilde{\lambda}_1\theta_{1c} - \theta_{1c}\widetilde{\lambda}_{12}\theta_{2c} - \theta_{2c}\widetilde{\lambda}_{21}\theta_{1c} - \theta_{2c}\widetilde{\lambda}_2\theta_{2c}. \tag{IA24}$$

It is then straightforward to characterize the equilibrium in closed form for the decentralized and centralized settings by extending the results in Propositions B.2 and B.3.

Figures IA.6 and IA.7 depict the aggregate investment positions and profits when there are only investors exploiting the first characteristic ($I_1 \geq 1$ and $I_2 = 0$) and there are investors exploiting both characteristics ($I_1 \geq 1$ and $I_2 \geq 1$), respectively. The figures compare the equilibria for four cases where the matrix of mean-reversion parameters is $\Phi = 0.1I_N$, $\Phi = 0.2I_N$, $\Phi = 0.3I_N$, and $\Phi = I_N$. We define persistent price-impact matrix C in (IA6) as $C = 2\Lambda$, where $\Lambda = E[\Lambda_t]$ is the unconditional expectation of the diagonal price-impact cost matrix defined in (3). Thus, according to Proposition IA.4.1 when matrix

Φ in (IA7) converges to the identity matrix, the model of persistent price-impact costs converges to the model of temporary price-impact costs in Lemma B.1. To estimate the price-impact diagonal matrix Λ_t, we rely on the results of Novy-Marx and Velikov (2016), who use trade-and-quote data to estimate λ_{tn} for the case with linear price impact and find that the R-squared of a cross-sectional regression of $\log(\lambda_{tn})$ on market capitalization is 70% and the slope is not statistically distinguishable from minus one. This suggests that a good approximation to the λ_{tn} of individual stocks is to assume that they are inversely proportional to the market capitalization of each firm. Moreover, Novy-Marx and Velikov (2016) find that the average price elasticity of supply, defined as the product of the price-impact parameter of the nth stock and its market capitalization at time t, $\lambda_{tn} \times me_{tn}$, is about 6.5. Based on this evidence, we estimate the stock-level price-impact parameter as $\lambda_{tn} = 6.5/me_{tn}$.

Our main observation is that the monotonicity properties of the investment positions and profits with respect to the number of investors for the case with persistent price impact are similar to those for the case with temporary price impact, and thus our insights are robust to considering persistent price impact. In particular, Figure IA.6 shows that, in the presence of persistent price impact, it continues to hold that as the number of investors exploiting the first characteristic increases, their aggregate investment position increases and their aggregate profit decreases. In other words, in the presence of persistent price impact it continues to hold that competition among investors exploiting the first characteristic erodes their profits. Moreover, Figure IA.7 shows that, in the presence of persistent price impact, it continues to hold that as the number of investors exploiting the second characteristic increases, the aggregate investment position and profits of investors exploiting the first characteristic increases. That is, in the presence of persistent price impact, it continues to hold that competition among investors exploiting the second characteristic increases the profits from the first characteristic.

Comparing the cases with persistent price impact ($\Phi = 0.1 I_N$, $\Phi = 0.2 I_N$, and $\Phi = 0.3 I_N$) with the case with temporary price impact ($\Phi = I_N$), we observe that the investment positions and profits from the first characteristic are larger in the presence of persistent price-impact costs than for purely temporary price-impact costs. This is because the persistent price-impact parameter for the first characteristic estimated from the data is smaller than its temporary counterpart ($\widetilde{\lambda}_1 < \lambda_1$). For the second characteristic, we obtain the opposite because $\widetilde{\lambda}_2 > \lambda_2$.

Figure IA.1: Competition with “book to market” as the single characteristic

This figure illustrates the effect of competition on aggregate investment positions and profits when there are only investors exploiting a single characteristic ($I_1 \geq 1$ and $I_2 = 0$). The figure depicts the aggregate investment position and profits for the first characteristic for the cases with $I_1 = 1, 2, 5, 10, 20, \infty$ investors. We consider the price-impact cost model of Frazzini et al. (2018) and use “book to market” as the single characteristic.

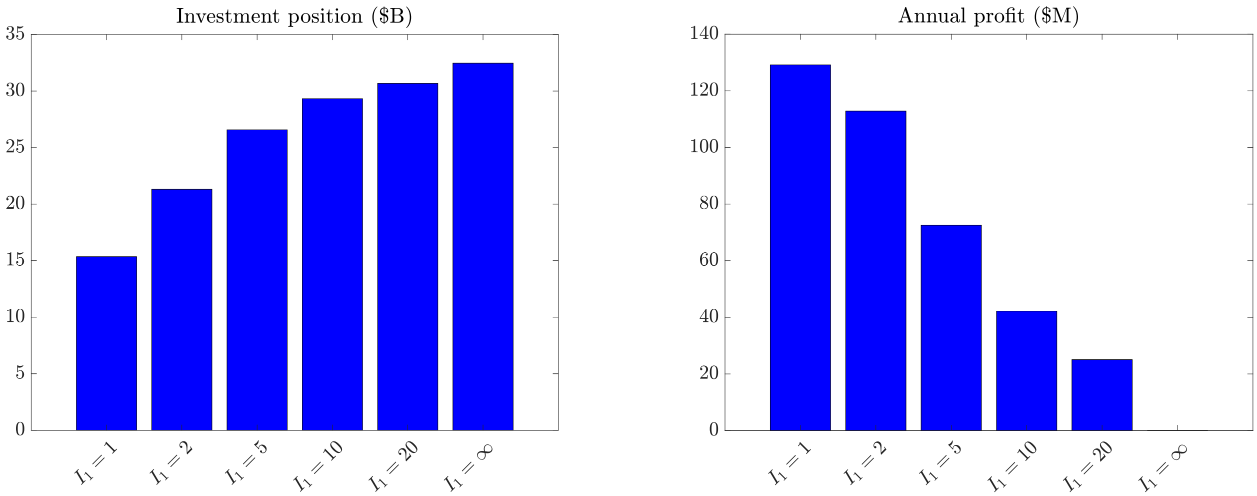
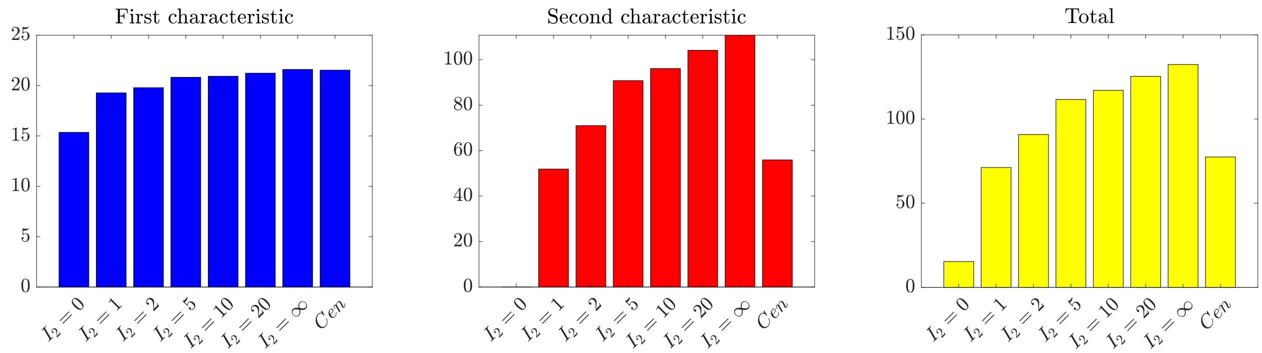


Figure IA.2: Competition with “book to market” and “gross profitability”

This figure depicts the investment positions and profits for the two characteristics for the decentralized setting where there is a single investor exploiting the first characteristic $I_1 = 1$ and $I_2 = 0, 1, 2, 5, 10, 20, \infty$ investors exploiting the second, and for the centralized setting (Cen). Panel (a) depicts the aggregate investment position in each of the two characteristics and the total investment position across both characteristics in billions of dollars. Panel (b) depicts the annual profits obtained from each characteristic and the total profits from both characteristics in millions of dollars. We consider the price-impact cost model of Frazzini et al. (2018) and use “book to market” as the first characteristic and “gross profitability” as the second.

(a) Aggregate investment positions ($B)



(b) Aggregate annual profits ($M)

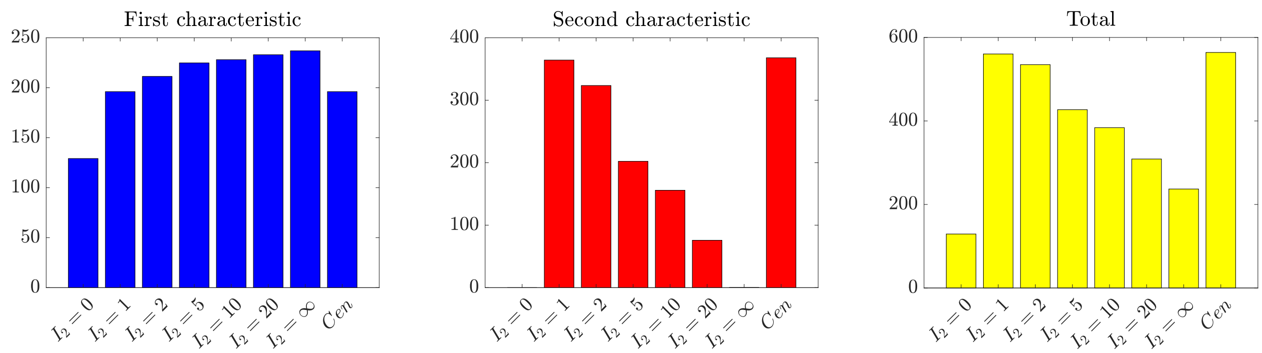


Figure IA.3: Competition with "momentum" as the single characteristic

This figure illustrates the effect of competition on aggregate investment positions and profits when there are only investors exploiting a single characteristic ($I_1 \geq 1$ and $I_2 = 0$). The figure depicts the aggregate investment position and profits for the first characteristic for the cases with $I_1 = 1, 2, 5, 10, 20, \infty$ investors. We consider the price-impact cost model of Frazzini et al. (2018) and use "momentum" as the single characteristic.

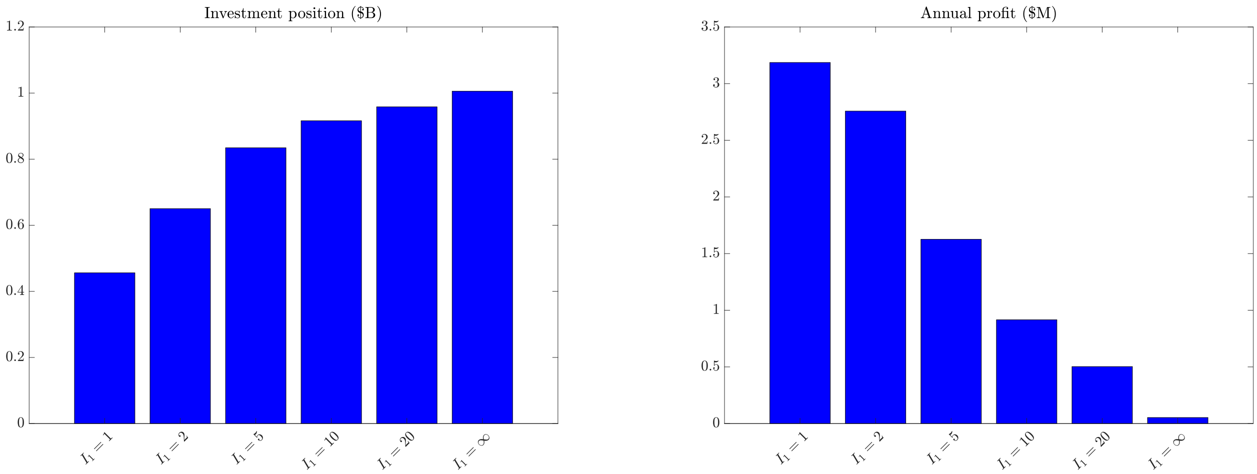
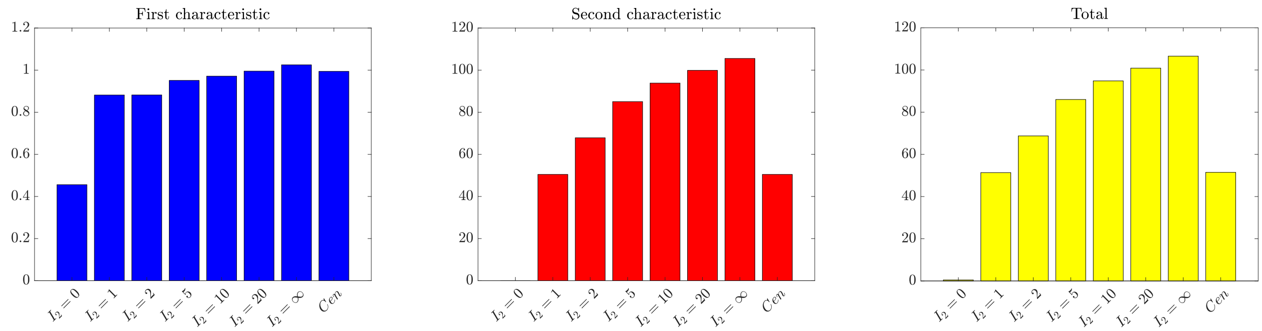


Figure IA.4: Competition with “momentum” and “gross profitability”

This figure depicts the investment positions and profits for the two characteristics for the decentralized setting where there is a single investor exploiting the first characteristic $I_1 = 1$ and $I_2 = 0, 1, 2, 5, 10, 20, \infty$ investors exploiting the second, and for the centralized setting (Cen). Panel (a) depicts the aggregate investment position in each of the two characteristics and the total investment position across both characteristics in billions of dollars. Panel (b) depicts the annual profits obtained from each characteristic and the total profits from both characteristics in millions of dollars. We consider the price-impact cost model of Frazzini et al. (2018) and use “momentum” as the first characteristic and “gross profitability” as the second.

(a) Aggregate investment positions ($B)



(b) Aggregate annual profits ($M)

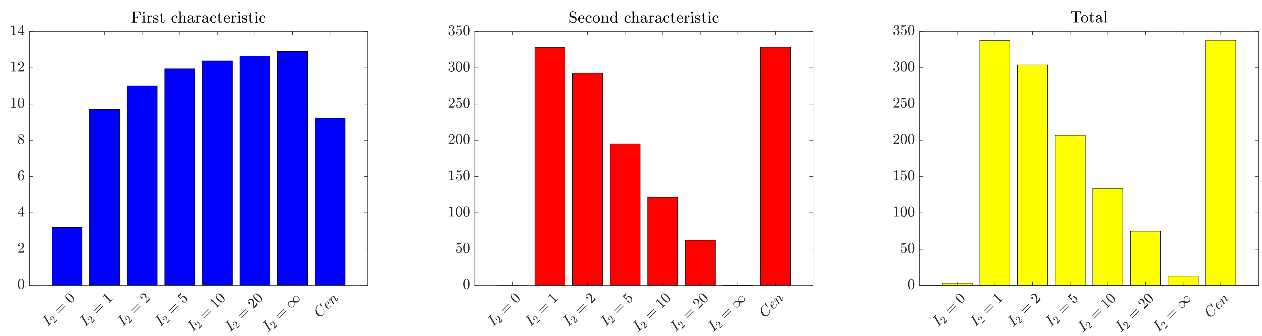
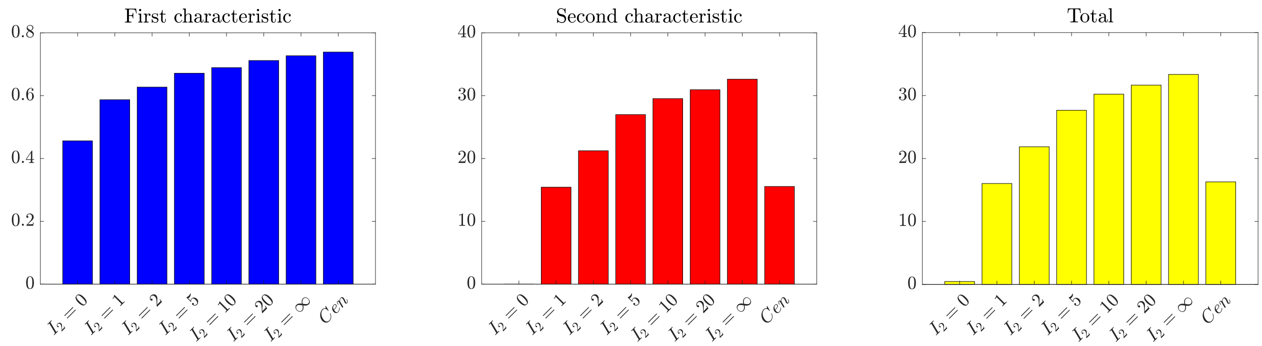


Figure IA.5: Competition with "momentum" and "book to market"

This figure depicts the investment positions and profits for the two characteristics for the decentralized setting where there is a single investor exploiting the first characteristic $I_1 = 1$ and $I_2 = 0, 1, 2, 5, 10, 20, \infty$ investors exploiting the second, and for the centralized setting (Cen). Panel (a) depicts the aggregate investment position in each of the two characteristics and the total investment position across both characteristics in billions of dollars. Panel (b) depicts the annual profits obtained from each characteristic and the total profits from both characteristics in millions of dollars. We consider the price-impact cost model of Frazzini et al. (2018) and use "momentum" as the first characteristic and "book to market" as the second.

(a) Aggregate investment positions ($B)



(b) Aggregate annual profits ($M)

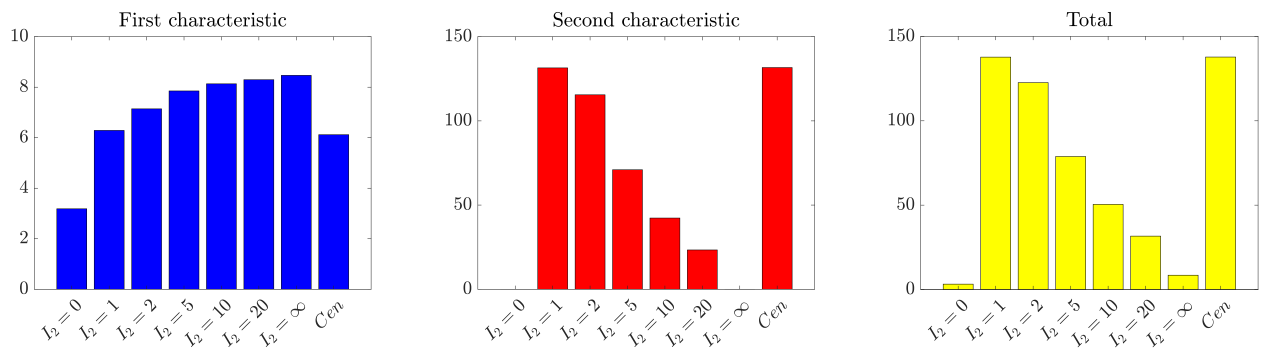


Figure IA.6: Competition in a single characteristic with persistent price impact

This figure illustrates the effect of competition in the presence of persistent linear price impact when there are no investors exploiting the second characteristic ($I_2 = 0$) and there are $I_1 = 1, 2, 5, 10, 20, \infty$ investors exploiting the first characteristic. We consider linear ($\alpha = 1.0$) and persistent price impact for four cases where the matrix of mean-reversion parameters are $\Phi = I_N$, $\Phi = 0.3I_N$, $\Phi = 0.2I_N$, and $\Phi = 0.1I_N$, where I_N is the N-dimensional identity matrix. We use "investment (asset growth)" as the first characteristic.

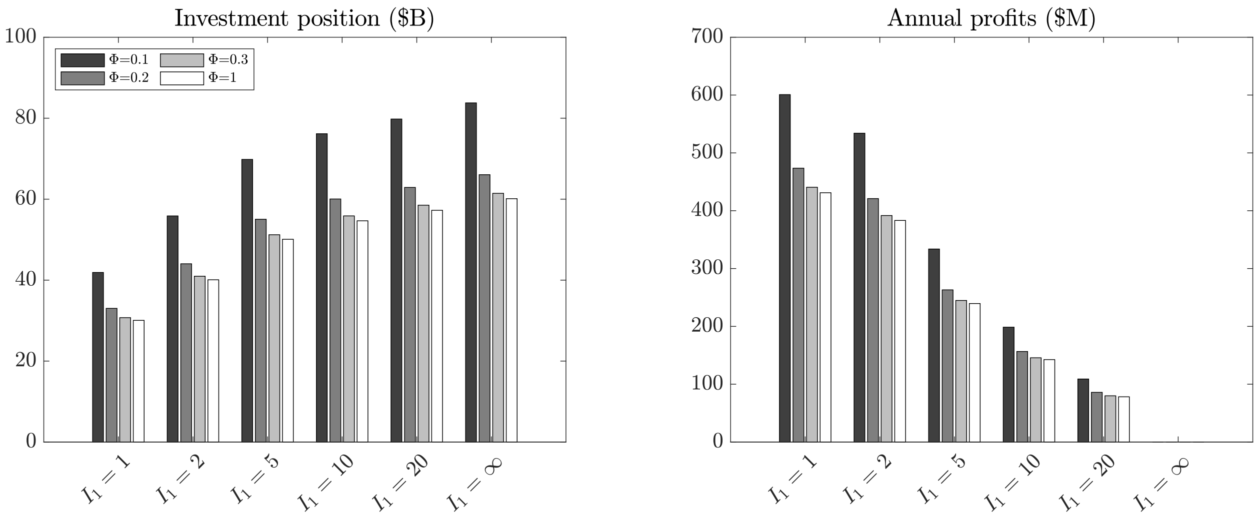
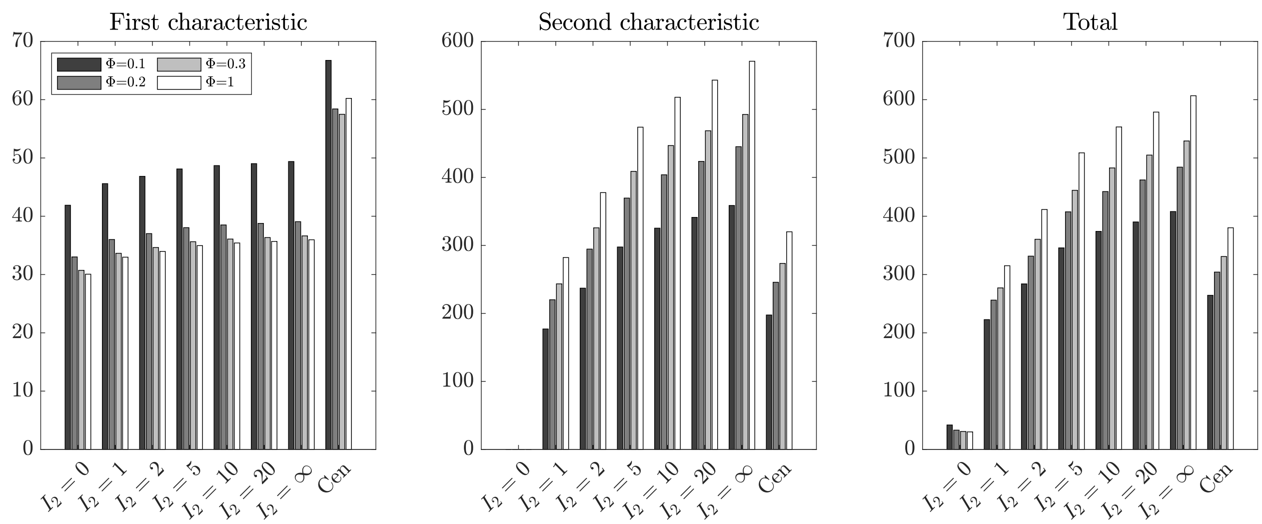


Figure IA.7: Competition in the second characteristic with persistent price impact

This figure depicts the investment positions and profits for the two characteristics in the presence of persistent linear price impact for the decentralized setting where there is a single investor exploiting the first characteristic $I_1 = 1$ and $I_2 = 0, 1, 2, 5, 10, 20, \infty$ investors exploiting the second, and for the centralized setting (Cen). Panel (a) depicts the equilibrium investment positions and Panel (b) the profits for the setting with linear ($\alpha = 1.0$) and persistent price impact for four cases where the matrix of mean-reversion parameters are $\Phi = I_N$, $\Phi = 0.3I_N$, $\Phi = 0.2I_N$, and $\Phi = 0.1I_N$, where I_N is the N-dimensional identity matrix. We use "investment (asset growth)" as the first characteristic and "gross profitability" as the second.

(a) Aggregate investment positions ($B)



(b) Aggregate profits ($M)

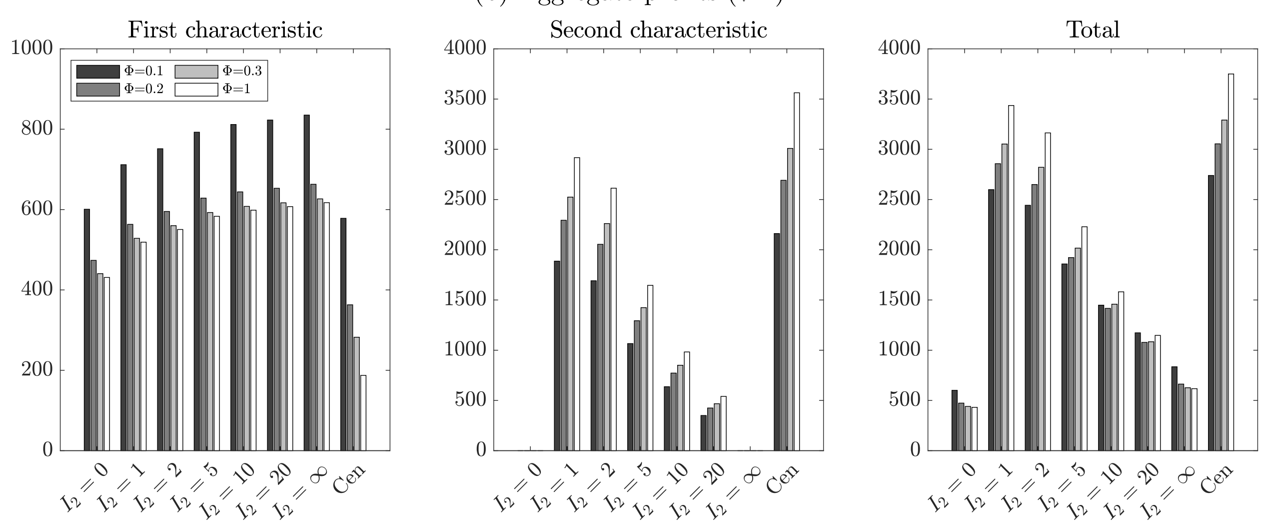


Table IA.1: Impact of trading diversification: first subsample

This table reports the capacity, optimal investment, and optimal annual profit of a single investor that exploits the 18 characteristics in isolation and in combination for the first half of our sample. For each characteristic, the first column reports its acronym and the remaining columns report its capacity, optimal investment, and optimal profit when considered in isolation and in combination, as well as the percentage increase in these quantities when the characteristic is considered in combination instead of in isolation. We obtain the optimal investment and profit by solving problem (9) for each of the 18 characteristics in isolation and in combination, with the price-impact cost PIC_t evaluated using the model of Frazzini et al. (2018) in Equation (13). The investment is given by the optimal value of θ and the annual profit is 12 times the optimal objective of problem (9). We express all quantities in terms of market capitalization at the end of our full sample (December 2018).

	Capacity			Investment			Profit		
Characteristic	Isol. ($bill.)	Comb. ($bill.)	Incr. (%)	Isol. ($bill.)	Comb. ($bill.)	Incr. (%)	Isol. ($mill.)	Comb. ($mill.)	Incr. (%)
gma	41.131	55.210	34	19.365	26.222	35	105.24	150.16	43
bm	11.845	18.859	59	5.527	8.957	62	37.84	68.55	81
herf	8.382	16.731	100	3.789	7.946	110	4.99	14.43	189
agr	7.475	12.339	65	3.475	5.860	69	32.06	60.65	89
rd_mve	10.801	10.670	−1	5.111	5.068	−1	60.14	61.95	3
chatoia	3.485	6.114	75	1.607	2.904	81	11.40	24.86	118
bm_ia	0.118	4.734	3905	0.051	2.248	4270	0.03	3.60	12285
mve	3.038	3.278	8	1.449	1.557	7	32.80	35.66	9
ps	2.310	2.643	14	1.059	1.255	19	5.29	7.92	50
beta	0.414	1.732	319	0.189	0.823	335	0.26	2.15	727
mom12m	0.813	1.587	95	0.368	0.754	105	3.73	9.78	162
chtx	0.499	1.102	121	0.228	0.523	130	1.21	4.02	233
sue	0.426	0.970	127	0.194	0.461	137	1.23	3.87	214
retvol	0.257	0.908	254	0.114	0.431	279	1.24	6.56	430
std_turn	0.000	0.365	-	0.000	0.174	-	0.00	0.56	-
mom1m	0.006	0.331	5392	0.003	0.157	5933	0.01	2.25	36707
zerotrade	0.036	0.323	787	0.016	0.154	845	0.08	1.64	2061
pchgm_pchsale	0.253	0.063	−75	0.112	0.030	−73	0.17	0.10	−40
Total	91.290	137.960	51	42.659	65.525	54	297.72	458.71	54

Table IA.2: Impact of trading diversification: second subsample

This table reports the capacity, optimal investment, and optimal annual profit of a single investor that exploits the 18 characteristics in isolation and in combination for the second half of our sample. For each characteristic, the first column reports its acronym and the remaining columns report its capacity, optimal investment, and optimal profit when considered in isolation and in combination, as well as the percentage increase in these quantities when the characteristic is considered in combination instead of in isolation. We obtain the optimal investment and profit by solving problem (9) for each of the 18 characteristics in isolation and in combination, with the price-impact cost PIC_t evaluated using the model of Frazzini et al. (2018) in Equation (13). The investment is given by the optimal value of θ and the annual profit is 12 times the optimal objective of problem (9). We express all quantities in terms of market capitalization at the end of our full sample (December 2018).

	Capacity			Investment			Profit		
Characteristic	Isol. ($bill.)	Comb. ($bill.)	Incr. (%)	Isol. ($bill.)	Comb. ($bill.)	Incr. (%)	Isol. ($mill.)	Comb. ($mill.)	Incr. (%)
gma	60.893	80.241	32	30.446	38.395	26	205.06	302.67	48
rd_mve	58.121	59.463	2	28.030	28.452	2	1246.73	1277.08	2
herf	23.337	47.972	106	10.795	22.954	113	43.09	101.84	136
bm	17.381	21.964	26	8.218	10.510	28	80.47	111.28	38
beta	14.011	18.620	33	6.506	8.910	37	53.02	76.64	45
agr	8.283	16.191	95	3.838	7.747	102	33.94	82.68	144
pchgm_pchsale	1.193	3.934	230	0.544	1.883	246	1.86	9.63	418
ps	1.581	3.272	107	0.725	1.566	116	3.68	11.04	200
mve	1.548	1.677	8	0.739	0.802	9	15.22	17.20	13
mom12m	0.173	1.572	808	0.078	0.752	869	0.30	5.62	1796
chatoia	0.000	−1.436	-	0.000	−0.687	-	0.00	−0.48	-
chtx	0.218	1.427	554	0.099	0.683	587	0.29	3.78	1218
sue	0.298	1.059	256	0.135	0.507	275	0.77	4.35	466
bm_ia	0.001	0.902	-	0.000	0.432	-	0.00	0.29	-
std_turn	0.000	0.844	-	0.000	0.404	-	0.00	1.41	-
retvol	0.000	0.764	-	0.001	0.366	58712	0.00	3.29	-
mom1m	0.000	0.239	-	0.000	0.115	-	0.00	0.87	-
zerotrade	0.000	0.184	-	0.000	0.088	-	0.00	0.22	-
Total	187.037	261.762	40	90.154	125.250	39	1684.42	2009.43	19